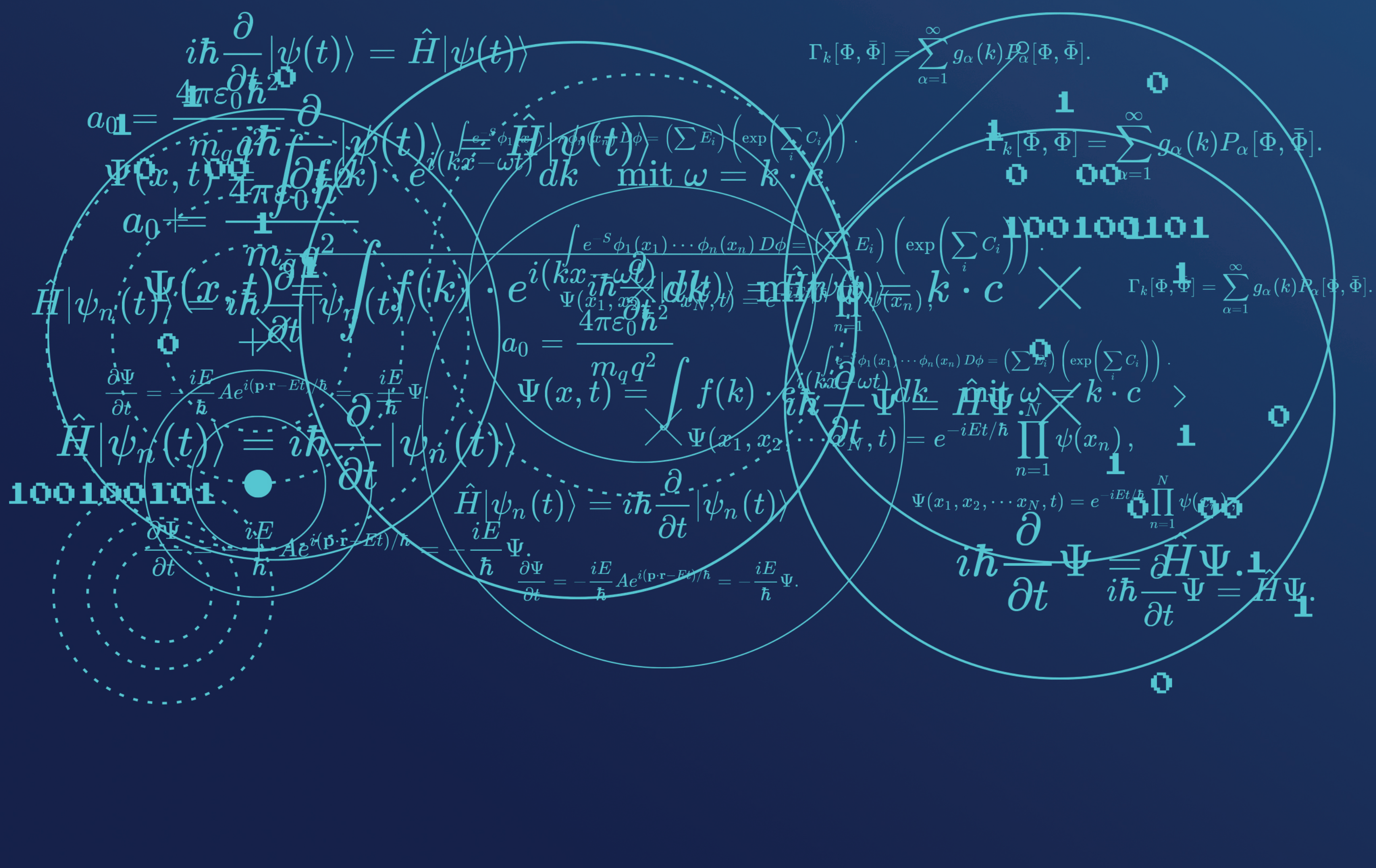
Open Quantum Physics and Environmental Heat Conversion into Usable Energy



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Open Quantum Physics and Environmental Heat Conversion into Usable Energy

(Volume 4)

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PREFACE

Some time ago, after a long career, I asked myself what quantum mechanics is. Starting an investigation with the basic laws of Plank-Einstein and de Broglie, I came to the conclusion that a quantum system is completely described only by two dynamic equations in the two conjugate spaces of the coordinates and momentum, and more than that, with the Lagrangian instead of the Hamiltonian as it is considered in the Schrödinger equation. With the Lagrangian in the time-dependent phase of the wave packet describing a quantum particle in the coordinate space for a certain energy, we obtain this phase as a function of the coordinate velocity, which leads to the equality of the wave/group velocities with this velocity. More than that, when the relativistic Lagrangian, as a function of the coordinate velocities, is considered, this equality remains. On this basis, the two wavefunctions in the two conjugate spaces can be considered as amplitudes of matter distributions. We obtain the mass quantization rule as the equality of the mass of a particle and as the integral of its density, with the mass as a dynamic characteristic in the time-dependent phases of its wavefunctions. In this way, quantum mechanics and general relativity became a single theory. In this framework, we obtain a cosmological model describing the main characteristics of our universe, such as the Big Bang, inflation, redshift, dark matter, and dark energy, which is in full agreement with the theory of relativity. These subjects have been approached in the third volume of this book.

In this book, we reconsider this new quantum-relativistic theory in the more general case of the field-dressed particles. For such a field, we obtain the Lorentz force and the Maxwell equations in general relativity. We obtain quantum dynamic equations and wavefunctions for a field-dressed particle-antiparticle system, entirely describing the relativistic effects. We use Dirac’s formalisms for quantum mechanics and for general relativity. We revise the Fermi golden rule with applications to quantum electrodynamics. In this theoretical framework, we consider the quark dynamics under the action of the four forces acting in nature and obtain a grand unified theory. In this way, we avoid the huge ontological and cosmological difficulties raised by the Schrödinger-Heisenberg description, which, however, remains a brilliant approximate description and formalism, perfect for the steady states and, for one hundred years, has been leading to the spectacular results of our civilization and today is used in very important application fields.

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DEDICATION

I dedicate this book to my family, Miorita, Radu, Mihaela, Alex, and Julia, and to all young people tending to really understanding the world they live in.

ABSTRACT

Fundamental subjects of quantum mechanics and general relativity are presented in a unitary framework. Based on the fundamental quantum laws of Planck-Einstein and De Broglie, a quantum particle is described by wave packets in the conjugate spaces of the coordinates and momentum. With the time-dependent phases proportional to the Lagrangian, the group velocities of these wave packets are in agreement with the fundamental Hamilton equations. When the relativistic Lagrangian, as a function of the metric tensor and the matter velocity field, is considered, the wave velocities are equal to the wavefunction coordinate velocity, which means that these waves describe the matter propagation. The equality of the integrals of the matter densities over the coordinate and momentum spaces, with the mass in the Lagrangian of the time-dependent phases, which describes the particle dynamics, represents the mass quantization rule. Describing the interaction of a quantum particle with the electromagnetic field by a modification of the particle dynamics determined by additional terms in the time-dependent phases, with an electric potential conjugated to time and a vector potential conjugated to the coordinates, Lorentz's force and Maxwell's equations are obtained. With Dirac's Hamiltonian and operators satisfying the Clifford algebra, dynamic equations similar to those used in the quantum field theory are obtained, but with an additional relativistic function, depending on the velocity and the matter-field momentum. We obtain particle and antiparticle wavefunctions describing matter and anti-mater distributions. Unlike the conventional Fermi's golden rule, in the new theory, the particle transitions are described by the Lagrangian matrix elements over the Lagrangian eigenstates and the densities of these states. Transition rates are obtained for the two possible processes, with the spin conservation or with the spin inversion. In this framework, we consider Dirac's formalism of general relativity, with the basic concepts of the Christoffel symbols, covariant derivative, scalar density and matter conservation, the geodesic dynamics, curvature tensor, Bianci equations, Ricci tensor, Einstein's gravitation law, and the Schwarzschild metric tensor. From the action integrals for the gravitational field, matter, electromagnetic field, and electric charge, we obtain the generalized Lorentz force and Maxwell equations for general relativity. It is shown that the gravitation equation is not modified by the electromagnetic field. For a black hole, the velocity and the acceleration of a particle are obtained. At the formation of a black hole, as a perfectly spherical object of matter gravitationally concentrated inside the Schwarzschild boundary, the central matter explodes, and the inside matter is carried out towards this boundary, reaching there only in an infinite time. Based on this model, we conceive our universe as a huge black hole, with its essential properties, such as the Big Bang, inflation, low large-scale density, redshift, quasi-inertial behavior of the distant bodies, dark matter, and dark energy, entirely explained by the general relativity. For a quantum particle in a gravitational wave, we obtained a rotation of the metric tensor perpendicular to the propagation direction of this wave, with the angular momentum 2, which we call the graviton spin, and a rotation of the particle matter, with a half-integer spin for Fermions and an integer spin for Bosons. We apply this theory to a two-particle and a particle-antiparticle collision, as well as a two-body decay of a quantum particle. In this framework, we also obtain a unitary description of the four forces acting in nature. A system of equations for the quark coordinates in a proton is obtained.

Keywords: Antiparticle, Black hole, Big bang, Bianci equations, Blue quark, Blue gluon, Covariant derivative, Clifford algebra, Contravariant coordinate, Christoffel symbol, Covariant coordinate, Curvature, Colour space, Dirac hamiltonian, Density of states, Dirac spin operators, Down quark, Einstein’s equation of gravitation, Fermi's golden rule, Feynman diagram, Four-vector, Flavour space, Group velocity, Geodesic equation, Graviton spin, Green quark, Green gluon, Gell-mann operators, Grand unified theory, Heisenberg picture, Hamiltonian, Schwarzschild metric tensor, Lorentz force, Lagrange equations, Lagrangian, Metric tensor, Maxwell equations, Nucleon, Pauli spin operators, Quantum electrodynamics, Quantum flavour-dynamics, Quantum chromodynamics, Redshift, Ricci tensor, Red quark, Red gluon, Schrödinger picture, Scalar potential, Spin, Spinor, Schwarzschild singularities, Schwarzschild boundary, Strong interaction, Two-body collision, Two-body decay, Time-space interval, The least action, Up quark, Vector potential, Vertex, Inflation, Vacuum impedance, Virtual photon, Wave packet, Wave velocity, Weak interaction.

CHAPTER 1

Introduction

Quantum mechanics is a fundamental theory that helps us understand the world we live in. However, the mechanisms described by this theory are difficult to understand due to the peculiar principles of Heisenberg's uncertainty and wave-particle duality. These principles lead to significant difficulties for physicists and philosophers, essentially by changing the fundamental notion of understanding. More than that, this theory seems to contradict the theory of relativity, which also stands at the basis of understanding our world. As quantum mechanics is based on the Schrödinger and Heisenberg pictures, essentially coming from the Planck-Einstein law of particles described by oscillations in time, with the de Broglie law of oscillations in the coordinate space obtained as a consequence, we avoid these difficulties by taking into account the two laws on the same footing, which are in agreement with the general theory of relativity.

Quantum mechanics describes important phenomena of atomic, nuclear, and solid-state physics, as well as quantum optics, mainly by the dynamic equations of the system of interest. However, the realistic phenomena can be properly described only in the framework of open quantum physics, where the interaction with the surroundings is taken into account [1]. This is a difficult job, and the initially tried empirical approach leads to violations of the quantum principles, such as the positivity of the density matrix, the uncertainty principle, and the zero-point motion. However, based on the axiomatic approach of the complete positivity of a dynamical map, Lindblad succeeded in obtaining a correct master equation [2], which became generally accepted by its successful application to deep inelastic collisions by Sandulescu and Scutaru [3]. This equation has been applied to numerous other physical systems of interest [4-11]. By a more physical approach, this equation has been reobtained by Alicki and Lendi from the dynamics of the total system of the system of interest and environment [12]. However, this equation, which provides a correct description of the quantum dynamics as a function of the system operators, has the deficiency of including unspecified phenomenological parameters. In the framework of a microscopic theory, we obtained a quantum master equation for a system of fermions in a complex dissipative environment of other fermions, bosons, and a free electromagnetic field [13-16]. On this basis, we discovered a new physical principle of a spontaneous entropy decrease, as a resonance effect, in a molecular system coupled with a coherent electromagnetic

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field, contrary to the second law of thermodynamics asserting that the entropy can only increase [17], and invented semiconductor devices converting the environmental heat into usable energy [18-23].

It is remarkable that, while the quantum system of interest is a part of our universe, also including other physical coordinates, our physical universe itself is only a part, a hypersurface of the total universe, also including other coordinates, where this hypersurface is curved according to general relativity [24]. While the coupling of a system of interest with the rest of the physical universe is described by the coefficients of the quantum master equation, the coupling of the physical universe with the total universe is also described by a set of coefficients, the metric tensor, and mass. In the following chapters of this paper, we show that quantum mechanics [25] and general relativity [26] can be understood as a unitary theory [27-32], where a wavefunction describes the mass density of a quantum particle in the two conjugate spaces of coordinates and momentum and not only the probability density of a punctual quantum entity in the coordinate space, as in the conventional quantum mechanics.

In Chapter 2, we describe a quantum particle as a distribution of matter in the two conjugate spaces of coordinates and momentum according to general relativity. In Chapter 3, we consider a field interacting with a quantum particle by a scalar potential conjugated to time and a vector potential conjugated to the coordinates as in the Aharonov-Bohm effect and obtain the Lorentz force and the Maxwell equations. In Chapter 4, we consider a quantum particle in an electromagnetic field with Dirac's Hamiltonian and Dirac's operators satisfying the Clifford algebra. We obtain a wavefunction in the coordinate space as a propagation operator, depending on the coordinates, momentum operator, and electromagnetic potential, applied to a time-dependent wavefunction as an integral over the momentum space. For the inverse momentum-dependent wavefunction, we also obtain a propagation operator, depending on momentum coordinate operators and the vector potential, applied to a time-dependent wavefunction as an integral over the coordinate space. From the corresponding dynamic equations in the coordinate and momentum spaces, which, compared to the similar equations of the quantum field theory [33, 34], include an additional relativistic function depending on the velocity, we obtain wavefunctions for a particle-antiparticle system, such as charged matter distributions in the electromagnetic field. In Chapter 5, we obtain the particle matter density as the diagonal element over the coordinate states of the density operator defined by the time-dependent states. The dynamic equation of the density operator takes a form depending on the particle Lagrangian instead of the Hamiltonian as in the conventional quantum equation. Thus, we reformulate Fermi's golden rule with transition matrix elements of the Lagrangian and the densities of the Lagrangian states. We obtain the transition rates of a quantum particle in a quasi-continuum of states for two possible processes: spin conservation or spin inversion. In Chapter 6, we consider Dirac's formalism of general relativity for a physical universe as a four-dimensional hypersurface in the total universe, including a larger number of coordinates [24]. With the action integrals for the gravitational field, mass, electromagnetic field, and electric charge in Chapter 7, we obtain a generalization of the Maxwell equations and Lorentz's force for general relativity. In Chapter 8, we consider a black hole as a large quantity of matter gravitationally concentrated inside its Schwarzschild boundary. We find that outside the black hole, the matter is attracted, but only up to a distance three times the Schwarzschild radius. Between this distance and the Schwarzschild radius, any object is repelled so that it reaches this radius only in an infinite time. At the formation moment of a black hole, an explosion of the central matter may arise, carrying out this matter towards the Schwarzschild boundary. At that moment, a central quantum particle is crushed up in the momentum space, exploding in the coordinate space. At the Schwarzschild boundary, a quantum particle is crushed up in the coordinate space, and with any fluctuation in this boundary, the quantum particle is brought back with a quasi-infinite momentum. In Chapter 9, we consider our universe as a large black hole in the infinite, everlasting universe. This way, its essential characteristics, such as Big Bang inflation, the small large-scale matter density, the quasi-inertial behavior of distant bodies, the redshift as a gravitational effect, dark matter, and dark energy, are unitarily described. In Chapter 10, from a harmonical oscillation of the coordinates, described by the covariant d'Alembert equation, under the action of a gravitational wave, we derive the gravitational wave equation for the metric tensor as a d'Alembert equation with total derivatives. From the wavefunction of a quantum particle, essentially depending on this tensor, we obtain the dynamic equation of this particle in a gravitational wave. For a metric tensor of the first order in the coordinates, the quantum particle is accelerated in the direction of propagation of the gravitational wave. For a metric tensor of the second order in the coordinates, the quantum particle oscillates in the direction of oscillation of the gravitational wave, perpendicularly to its direction of propagation. For a first-order metric tensor, we obtain an invariant describing a rotation of this tensor perpendicularly to the direction of propagation with spin 2, which we call the graviton spin. We also obtain a matter rotation perpendicularly to the direction of propagation that we call the particle spin, with a half-integer value for a particle that is called a fermion and an integer spin for a particle that is called a boson. In Chapter 11, we consider applications to quantum electrodynamics. We obtain explicit expressions of the transition rates for two-body collisions and decays. In Chapter 12, we consider the four fields acting in nature: the gravitational field as a

CHAPTER 2

Quantum Particle Distributions of Matter

THE FUNDAMENTAL EINSTEIN AND DE BROGLIE EQUATIONS AND THE SCHRÖDINGER DYNAMIC EQUATION

As it is well-known, quantum mechanics is based on the Planck-Einstein empirical theory, asserting that matter is composed of harmonic oscillators, with the universal constant $h = 2\pi\hbar$ as a proportionality coefficient of the energy $H\left(\vec{p},\vec{r}\right) = E$ with the frequency $\omega = 2\pi\nu$,

$$E = h\nu = \hbar\omega \,, \tag{2.1}$$

which are described by wavefunctions of the form

$$\psi_E = e^{-\mathrm{i}2\pi\nu t} = e^{-\mathrm{i}\omega t} = e^{-\frac{\mathrm{i}}{\hbar}Et} = e^{-\frac{\mathrm{i}}{\hbar}Ht} . \tag{2.2}$$

According to the de Broglie empirical law, a quantum particle in motion with a momentum $\vec{p}$ is described by a wavefunction, with this momentum proportional to the wave vector,

$$\vec{p} = \hbar\vec{k} \,, \tag{2.3}$$

which is of the form

$$\psi_{\vec{p}} = e^{\mathrm{i}\vec{k}\vec{r}} = e^{\frac{\mathrm{i}}{\hbar}\vec{p}\vec{r}} . \tag{2.4}$$

Thus, for a free quantum particle, from the wavefunctions (2.2) and (2.4), we obtain a wavefunction

$$\psi = \psi_{\vec{p}}\psi_E = e^{\frac{\mathrm{i}}{\hbar}\left(\vec{p}\vec{r}-Ht\right)} , \tag{2.5}$$

describing a propagation with the velocity

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$$\frac{\partial\vec{r}}{\partial t}=\frac{\partial H}{\partial\vec{p}}. \tag{2.6}$$

From the wavefunction (2.5), we obtain the Hamiltonian operator

$$H=\mathrm{i}\hbar\frac{\partial}{\partial t}, \tag{2.7}$$

and the momentum operator

$$\vec{p}=-\mathrm{i}\hbar\frac{\partial}{\partial\vec{r}}. \tag{2.8}$$

From the wavefunction (2.5) with the Hamiltonian operator (2.7), we obtain the Schrödinger famous equation standing on the basis of the conventional quantum theory,

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi\left(\vec{r},t\right)=H\left(\vec{r},\vec{p}\right)\psi\left(\vec{r},t\right). \tag{2.9}$$

In the classical approximation, the Hamiltonian

$$H\left(\vec{r},\vec{p}\right)=T\left(\vec{p}\right)+U\left(\vec{r}\right)=\frac{\vec{p}^2}{2M}+U\left(\vec{r}\right), \tag{2.10}$$

and the momentum operator (2.8), the Schrödinger equation (2.9) takes the explicit form

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi\left(\vec{r},t\right)=\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial\vec{r}^2}+U\left(\vec{r}\right)\right]\psi\left(\vec{r},t\right), \tag{2.11}$$

which, in a steady state with an energy E, is

$$H\left(\vec{r},\vec{p}\right)\psi\left(\vec{r}\right)=\left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial\vec{r}^2}+U\left(\vec{r}\right)\right]\psi\left(\vec{r}\right)=E\psi\left(\vec{r}\right). \tag{2.12}$$

This equation stands at the basis of important application fields of atomic, nuclear, and solid-state structures, matter-field systems, and recent developments in open systems [1-23]. With the general solution of the Schrödinger equation (2.11),

$$\left|\psi(\vec{r},t)\right\rangle = e^{-\frac{\mathrm{i}}{\hbar}H(\vec{r},\vec{p})t}\left|\psi_0(\vec{r})\right\rangle \tag{2.13}$$

from the matrix elements of an arbitrary operator A,

$$\begin{aligned}\left\langle\phi(\vec{r},t)\right|A\left|\psi(\vec{r},t)\right\rangle &= \left\langle\phi_0(\vec{r})\right|e^{\frac{\mathrm{i}}{\hbar}H(\vec{r},\vec{p})t}Ae^{-\frac{\mathrm{i}}{\hbar}H(\vec{r},\vec{p})t}\left|\psi_0(\vec{r})\right\rangle \\ &= \left\langle\phi_0(\vec{r})\right|A(t)\left|\psi_0(\vec{r})\right\rangle.\end{aligned}$$

One obtains the time-dependent Heisenberg operator

$$A(t) = e^{\frac{\mathrm{i}}{\hbar}H(\vec{r},\vec{p})t}Ae^{-\frac{\mathrm{i}}{\hbar}H(\vec{r},\vec{p})t}, \tag{2.14}$$

which satisfies the Heisenberg equation

$$\frac{\partial}{\partial t}A(t) = \frac{\mathrm{i}}{\hbar}\left[H(\vec{r},\vec{p}),A(t)\right]. \tag{2.15}$$

From this equation for the coordinates and momentum operators, with the commutation relation

$$\begin{aligned}\left[\vec{r},\vec{p}\right] &= -\mathrm{i}\hbar\vec{r}\frac{\partial}{\partial\vec{r}} + \mathrm{i}\hbar\frac{\partial}{\partial\vec{r}}\vec{r} = -\mathrm{i}\hbar\vec{r}\frac{\partial}{\partial\vec{r}} + \mathrm{i}\hbar\vec{r}\frac{\partial}{\partial\vec{r}} + \mathrm{i}\hbar\frac{\partial\vec{r}}{\partial\vec{r}} \\ &= \mathrm{i}\hbar\left(\vec{1}_x\frac{\partial}{\partial x} + \vec{1}_y\frac{\partial}{\partial y} + \vec{1}_z\frac{\partial}{\partial z}\right)\left(\vec{1}_x x + \vec{1}_y y + \vec{1}_z z\right) = 3\mathrm{i}\hbar.\end{aligned} \tag{2.16}$$

We obtain the mean-value equations

$$\begin{aligned}\frac{\partial}{\partial t}\left\langle\vec{r}(t)\right\rangle &= \frac{\mathrm{i}}{\hbar}\left\langle\left[\frac{\vec{p}^2}{2M} + U(\vec{r}),\vec{r}\right]\right\rangle = \frac{\mathrm{i}}{2M\hbar}\left\langle\vec{p}\left[\vec{p},\vec{r}\right] + \left[\vec{p},\vec{r}\right]\vec{p}\right\rangle = 3\frac{\left\langle\vec{p}(t)\right\rangle}{M} \\ \frac{\partial}{\partial t}\left\langle\vec{p}(t)\right\rangle &= \frac{\mathrm{i}}{\hbar}\left\langle\left[\frac{\vec{p}^2}{2M} + U(\vec{r}),\vec{p}\right]\right\rangle = \frac{\mathrm{i}}{\hbar}\left\langle\left[U(\vec{r}),-\mathrm{i}\hbar\frac{\partial}{\partial\vec{r}}\right]\right\rangle \\ &= \frac{\mathrm{i}}{\hbar}\left\langle -\mathrm{i}\hbar U(\vec{r})\frac{\partial}{\partial\vec{r}} + \mathrm{i}\hbar\frac{\partial}{\partial\vec{r}}U(\vec{r})\right\rangle \\ &= \frac{\mathrm{i}}{\hbar}\left\langle -\mathrm{i}\hbar U(\vec{r})\frac{\partial}{\partial\vec{r}} + \mathrm{i}\hbar\frac{\partial U(\vec{r})}{\partial\vec{r}} + \mathrm{i}\hbar U(\vec{r})\frac{\partial}{\partial\vec{r}}\right\rangle = -\left\langle\frac{\partial U(\vec{r})}{\partial\vec{r}}\right\rangle,\end{aligned} \tag{2.17}$$

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CHAPTER 3

Electromagnetic Field

QUANTUM PARTICLES IN THE ELECTROMAGNETIC FIELD AND THE PARTICLE-FIELD HAMILTONIAN

Since the electromagnetic field is much stronger than the gravitational one, we consider a charged quantum particle in a flat space, with an electric charge e interacting with an electromagnetic field with an electric potential $U(\vec{r})$ and a vector potential $\vec{A}(\vec{r},t)$, described by wavefunctions of the form (2.43),

$$\begin{aligned}\psi(\vec{r},t) &= \frac{1}{(2\pi\hbar)^{3/2}}\int \varphi(\vec{P},t)e^{\frac{\mathrm{i}}{\hbar}\left[\vec{P}\vec{r}-L(\vec{r},\dot{\vec{r}},t)t\right]}\mathrm{d}^3\vec{P} \\ \varphi(\vec{P},t) &= \frac{1}{(2\pi\hbar)^{3/2}}\int \psi(\vec{r},t)e^{-\frac{\mathrm{i}}{\hbar}\left[\vec{P}\vec{r}-L(\vec{r},\dot{\vec{r}},t)t\right]}\mathrm{d}^3\vec{r},\end{aligned} \tag{3.1}$$

with a Lagrangian of the form (2.41), for the electric potential $U(\vec{r})$ conjugated to time, and the vector potential $\vec{A}(\vec{r},t)$ conjugated to the coordinates, as in the Aharonov-Bohm effect,

$$L(\vec{r},\dot{\vec{r}},t) = -Mc^2\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}} - eU(\vec{r}) + e\vec{A}(\vec{r},t)\dot{\vec{r}} \; . \tag{3.2}$$

In this case, the canonical matter field momentum

$$\vec{P} = \frac{\partial}{\partial\dot{\vec{r}}}L(\vec{r},\dot{\vec{r}},t) = \frac{M\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}} + e\vec{A}(\vec{r},t) = \vec{p} + e\vec{A}(\vec{r},t), \tag{3.3}$$

is the sum of the electromagnetic momentum $e\vec{A}(\vec{r},t)$ with the mechanical momentum

$$\vec{p}=\frac{M\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^{2}}{c^{2}}}}\ . \tag{3.4}$$

With these expressions, we obtain the matter-field Hamiltonian

$$\begin{aligned} H\left(\vec{P},\vec{r}\right)&=\vec{P}\,\dot{\vec{r}}-L\left(\vec{r},\dot{\vec{r}},t\right)\\ &=\left[\frac{M\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^{2}}{c^{2}}}}+\cancel{e\vec{A}\left(\vec{r},t\right)}\right]\dot{\vec{r}}-\left[-\mathrm{M}\mathrm{c}^{2}\sqrt{1-\frac{\dot{\vec{r}}^{2}}{c^{2}}}-eU\left(\vec{r}\right)+\cancel{e\vec{A}\left(\vec{r},t\right)\dot{\vec{r}}}\right]\\ &=\frac{Mc^{2}}{\sqrt{1-\frac{\dot{\vec{r}}^{2}}{c^{2}}}}+eU\left(\vec{r}\right)=E_{m}+U_{e}=c\tilde{E}+U_{e}=E, \end{aligned} \tag{3.5}$$

with the energy formed from the mechanical energy, proportional to the rest energy Mc^{2} and Lorentz's factor,

$$E_{m}=\frac{Mc^{2}}{\sqrt{1-\frac{\dot{\vec{r}}^{2}}{c^{2}}}}, \tag{3.6}$$

and the potential energy of the particle in the electromagnetic field

$$U_{e}\left(\vec{r}\right)=eU\left(\vec{r}\right), \tag{3.7}$$

where, for the convenience of further calculations, we used the notation $\tilde{E},\ E_{m}=c\tilde{E}$, that we call the normalized energy, which in fact, has the dimension of a momentum. To obtain the canonical expression of the Hamiltonian (3.5), from (3.4), we calculate the normalized velocity as a function of the momentum,

$$\frac{\dot{\vec{r}}}{c}=\frac{\vec{p}}{\sqrt{M^{2}c^{2}+\vec{p}^{2}}}, \tag{3.8}$$

as the denominator of the Hamiltonian (3.5) is

$$\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}=\frac{Mc}{\sqrt{M^2c^2+\vec{p}^2}}\ . \qquad \textbf{(3.9)}$$

We obtain the canonical expression of this Hamiltonian,

$$\begin{aligned} H\left(\vec{P},\vec{r}\right) &\doteq c\sqrt{M^2c^2+\vec{p}^2}+eU\left(\vec{r}\right) \\ &= c\sqrt{M^2c^2+\left[\vec{P}-e\vec{A}(\vec{r},t)\right]^2}+eU\left(\vec{r}\right) \\ &= c\tilde{H}\left(\vec{P},\vec{r}\right)+eU\left(\vec{r}\right)\doteq c\tilde{E}\left(\vec{P}\right)+eU\left(\vec{r}\right)=E, \end{aligned} \qquad \textbf{(3.10)}$$

where we used the notations $\tilde{H}\left(\vec{P},\vec{r}\right)$ for the normalized Hamiltonian, and $\tilde{E}$ for the normalized energy. From the conservation of the energy E, which means a constant term

$$\left[\vec{P}-e\vec{A}(\vec{r},t)\right]^2=\vec{P}^2-2e\vec{P}\vec{A}(\vec{r},t)+e^2\vec{A}^2\left(\vec{r},t\right)=\text{const}\ ,$$

we obtain a conservative canonical matter-field momentum $\vec{P}$ perpendicular to the vector potential $\vec{A}(\vec{r},t)$ as a vector rotating with a constant amplitude $\left|\vec{A}(\vec{r},t)\right|$,

$$\vec{P}\vec{A}(\vec{r},t)=0, \qquad \left|\vec{A}(\vec{r},t)\right|=\text{const}\ . \qquad (3.11)$$

At the same time, from the conservation of the canonical matter-field momentum (3.3), we notice that the particle momentum $\vec{p}$, of the quantum particle in the electromagnetic field includes a rotational component $-e\vec{A}(\vec{r},t)$ perpendicular to the translational component $\vec{P}$. With the Hamiltonian (3.10), the wavefunctions (3.1) of a quantum particle in an electromagnetic field are

CHAPTER 4

Quantum Particles in an Electromagnetic Field

QUANTUM PARTICLE WAVEFUNCTIONS IN ELECTROMAGNETIC FIELD AND DYNAMIC EQUATIONS

We consider a quantum particle in an electromagnetic field with the wavefunction (3.12)

$$\begin{aligned} \psi(\vec{r},t) &= \frac{1}{(2\pi\hbar)^{3/2}} \int e^{\frac{i}{\hbar}\left\{\vec{P}\cdot\vec{r} - \left[\vec{P}\dot{\vec{r}} - H(\vec{P},\vec{r})\right]t\right\}} \varphi(\vec{P},t)\,\mathrm{d}^3\vec{P} \\ \varphi(\vec{P},t) &= \frac{1}{(2\pi\hbar)^{3/2}} \int e^{-\frac{i}{\hbar}\left\{\vec{P}\cdot\vec{r} - \left[\vec{P}\dot{\vec{r}} - H(\vec{P},\vec{r})\right]t\right\}} \psi(\vec{r},t)\,\mathrm{d}^3\vec{r}, \end{aligned} \tag{4.1}$$

as functions of the matter-field Hamiltonian (3.10), with Dirac's relativistic Hamiltonian and the canonical matter-field momentum (3.3),

$$\begin{aligned} H(\vec{P},\vec{r}) &= c\tilde{H} + eU(\vec{r}) = c\sqrt{M^2c^2 + \vec{p}^2} + eU(\vec{r}) = c\sqrt{M^2c^2 + \left[\vec{P} - e\vec{A}(\vec{r},t)\right]^2} + eU(\vec{r}) \\ &= \alpha_0 Mc^2 + c\alpha_j p^j + eU(\vec{r}) = \alpha_0 Mc^2 + eU(\vec{r}) + c\alpha_j\left[P^j - eA^j(\vec{r},t)\right], \end{aligned} \tag{4.2}$$

depending on the Dirac spin operators α_μ, which satisfy the anticommutation relations

$$\left\{\alpha_\mu, \alpha_\nu\right\} = 2\delta_{\mu\nu}, \qquad \alpha_\mu{}^2 = 1. \tag{4.3}$$

For a free quantum particle, we consider the momentum four-vector $p^\mu = \left(p^0 = \tilde{H}, p^j\right)$, of the invariant length of the rest momentum,

$$Mc = \sqrt{\tilde{H}^2 - \vec{p}^2} = \sqrt{p^\mu p_\mu} = \sqrt{g^{\mu\nu} p_\mu p_\nu}\,\hat{\hat{1}} = \gamma^\mu p_\mu, \tag{4.4}$$

on the Dirac operators satisfying the Clifford algebra

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$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=2g^{\mu\nu}, \tag{4.5}$$

which, for a flat space,

$$g^{00}=1,\quad g^{11}=g^{22}=g^{33}=-1,\quad g^{\mu\nu}\Big|_{\mu\neq\nu}=0, \tag{4.6}$$

is

$$\left\{\gamma^{0},\gamma^{i}\right\}=0,\quad \left\{\gamma^{i},\gamma^{j}\right\}=0,\quad \gamma^{0^2}=g^{00}=1,\quad \gamma^{i^2}=g^{ii}=-1. \tag{4.7}$$

It is remarkable that the operators γ are functions of the operators α,

$$\gamma^{0}=\alpha_0,\quad \gamma^{i}=\alpha_0\alpha_i,\quad \gamma^{i\dagger}=\alpha_i\alpha_0=-\gamma^{i}, \tag{4.8}$$

which with the algebra (4.3) leads to the Clifford algebra (4.7),

$$\begin{aligned}
&\gamma^{0}\gamma^{i}=\alpha_0\alpha_0\alpha_i=\alpha_i,\quad \gamma^{i}\gamma^{0}=\alpha_0\alpha_i\alpha_0=-\alpha_0\alpha_0\alpha_i=-\alpha_i \quad \Rightarrow\ \left\{\gamma^{0},\gamma^{i}\right\}=0\\
&\gamma^{i}\gamma^{j}=\alpha_0\alpha_i\alpha_0\alpha_j=-\alpha_i\alpha_0\alpha_0\alpha_j=-\mathrm{i}\varepsilon_{ijk}\alpha_k, \quad \Rightarrow\ \left\{\gamma^{i},\gamma^{j}\right\}=0\\
&\gamma^{0^2}=\alpha_0{}^2=1=g^{00},\quad \gamma^{i^2}=\alpha_0\alpha_i\alpha_0\alpha_i=-\alpha_i\alpha_0\alpha_0\alpha_i=-\alpha_i{}^2=-1=g^{ii}.
\end{aligned} \tag{4.9}$$

With the Hamiltonian (4.2), the particle wavefunction (4.1) in the coordinate space takes the form

$$\begin{aligned}
\psi\left(\vec r,t\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int e^{\frac{\mathrm{i}}{\hbar}\left\{\vec P\cdot\vec r-\left[\vec P\dot{\vec r}-\left(\alpha_0Mc^2+eU(\vec r)+c\alpha_j\left(P^j-eA^j(\vec r,t)\right)\right)\right]t\right\}}\varphi\left(\vec P,t\right)\mathrm{d}^3\vec P\\
&=e^{\frac{\mathrm{i}}{\hbar}\left\{\vec P\cdot\vec r-\left[-eU(\vec r)+ec\alpha_jA^j(\vec r,t)\right]t\right\}}\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int e^{-\frac{\mathrm{i}}{\hbar}\left[\vec P\dot{\vec r}-\left(\alpha_0Mc^2+c\alpha_jP^j\right)\right]t}\varphi\left(\vec P,t\right)\mathrm{d}^3\vec P\\
&=e^{\frac{\mathrm{i}}{\hbar}\left\{\vec P\cdot\vec r-\left[-eU(\vec r)+ec\alpha_jA^j(\vec r,t)\right]t\right\}}\psi_t\left(\vec r,t\right)=\mathcal{P}_f\psi_t\left(\vec r,t\right),
\end{aligned} \tag{4.10}$$

as a time-dependent wavefunction

$$\psi_t\left(\vec r,t\right)=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int e^{-\frac{\mathrm{i}}{\hbar}\left[\vec P\dot{\vec r}-\left(\alpha_0Mc^2+c\alpha_jP^j\right)\right]t}\varphi\left(\vec P,t\right)\mathrm{d}^3\vec P, \tag{4.11}$$

with a propagation operator of the particle in the electromagnetic field, depending on the coordinates,

$$\mathcal{P}_f = e^{\frac{\mathrm{i}}{\hbar}\left\{\vec{P}\cdot\vec{r}-\left[-eU(\vec{r})+ec\alpha_j A^j(\vec{r},t)\right]t\right\}} = e^{\frac{\mathrm{i}}{\hbar}\left\{\left[\vec{p}+e\vec{A}(\vec{r},t)\right]\cdot\vec{r}-\left[-eU(\vec{r})+ec\alpha_j A^j(\vec{r},t)\right]t\right\}} = \mathcal{F}\mathcal{P} \tag{4.12}$$

as the product of the particle propagation operator

$$\mathcal{P} = e^{\frac{\mathrm{i}}{\hbar}\vec{p}\cdot\vec{r}}, \tag{4.13}$$

with the field propagation operator

$$\mathcal{F} = e^{\frac{\mathrm{i}}{\hbar}\left\{e\vec{A}(\vec{r},t)\cdot\vec{r}-\left[-eU(\vec{r})+ec\alpha_j A^j(\vec{r},t)\right]t\right\}}. \tag{4.14}$$

With the Hamiltonian (4.2), the wavefunction (4.1) in the momentum space with the wavefunction (4.10) in the coordinate space,

$$\begin{aligned}
\varphi\left(\vec{P},t\right) &= \frac{1}{(2\pi\hbar)^{3/2}} e^{-\frac{\mathrm{i}}{\hbar}\vec{P}\cdot\vec{r}} \int e^{\frac{\mathrm{i}}{\hbar}\left[\vec{P}\dot{\vec{r}}-H\left(\vec{P},\vec{r},t\right)\right]t} \psi(\vec{r},t)\,\mathrm{d}^3\vec{r} \\
&= \frac{1}{(2\pi\hbar)^{3/2}} e^{-\frac{\mathrm{i}}{\hbar}\vec{P}\cdot\vec{r}} \int \begin{bmatrix} e^{\frac{\mathrm{i}}{\hbar}\left\{\vec{P}\dot{\vec{r}}-\left[\alpha_0 Mc^2+\underline{eU(\vec{r})}+c\alpha_j\left(P^j-\underline{\underline{eA^j(\vec{r},t)}}\right)\right]\right\}t} \\ e^{\frac{\mathrm{i}}{\hbar}\left\{\vec{P}\cdot\vec{r}-\left[-\underline{eU(\vec{r})}+\underline{\underline{ec\alpha_j A^j(\vec{r},t)}}\right]t\right\}}\psi_t(\vec{r},t) \end{bmatrix} \mathrm{d}^3\vec{r} \\
&= \frac{1}{(2\pi\hbar)^{3/2}} e^{-\frac{\mathrm{i}}{\hbar}\vec{P}\cdot\vec{r}} \int e^{\frac{\mathrm{i}}{\hbar}\left[\vec{P}\dot{\vec{r}}-\left(\alpha_0 Mc^2+c\alpha_j P^j\right)\right]t} e^{\frac{\mathrm{i}}{\hbar}\vec{P}\cdot\vec{r}} \psi_t(\vec{r},t)\,\mathrm{d}^3\vec{r},
\end{aligned} \tag{4.15}$$

takes the form

$$\varphi\left(\vec{P},t\right) = e^{-\frac{\mathrm{i}}{\hbar}\vec{P}\cdot\vec{r}} \varphi_t\left(\vec{P},t\right) = e^{-\frac{\mathrm{i}}{\hbar}\left[\vec{p}+e\vec{A}(\vec{r},t)\right]\cdot\vec{r}} \varphi_t\left(\vec{P},t\right) = \mathcal{P}_{mf}{}^{-1}\varphi_t\left(\vec{P},t\right), \tag{4.16}$$

of a time-dependent wavefunction

CHAPTER 5

Quantum Particle Transitions in the Electromagnetic Field and Fermi's Golden Rule

THE DENSITY MATRIX OF A QUANTUM PARTICLE AS A DISTRIBUTION OF MATTER

The mass (2.38) of a quantum particle, in an electromagnetic field with a wavefunction (4.10), is obtained as an integral of the matter density

$$\rho_M(\vec{r},t) = M\left|\psi(\vec{r},t)\right|^2 = M\psi_t^{\dagger}(\vec{r},t)\mathcal{P}_f^{-1}\mathcal{P}_f\psi_t(\vec{r},t) = M\psi_t^{\dagger}(\vec{r},t)\psi_t(\vec{r},t) \\ = M\rho(\vec{r},t), \tag{5.1}$$

which is the product of the mass with a real density function, as the length of the time-dependent wavefunction four-vector

$$\rho(\vec{r},t) = \psi_t^{\dagger}(\vec{r},t)\psi_t(\vec{r},t). \tag{5.2}$$

This function takes the form of a diagonal matrix element over the coordinate states $|\vec{r}\rangle$,

$$\rho(\vec{r},t) = \psi_t^{\dagger}(\vec{r},t)\psi_t(\vec{r},t) = \psi_t^{*\dagger}(\vec{r},t)\psi_t^{*}(\vec{r},t) = \langle\vec{r}|\psi_t\rangle\langle\psi_t|\vec{r}\rangle \\ = \langle\vec{r}|\rho(t)|\vec{r}\rangle, \tag{5.3}$$

of the density operator

$$\rho(t) = |\psi_t\rangle\langle\psi_t|. \tag{5.4}$$

We consider a quantum particle in an electromagnetic field, with a time-dependent wavefunction satisfying the dynamic equation of the form (4.21) in the coordinate space, with a Lagrangian $L(\vec{P},\vec{r})$ and a Hamiltonian $H(\vec{P},\vec{r})$,

$$\frac{\partial}{\partial t}\langle\vec{r}|\psi_t(t)\rangle = -\frac{i}{\hbar}L(\vec{P},\vec{r})\langle\vec{r}|\psi_t(t)\rangle = -\frac{i}{\hbar}\left[\vec{P}\dot{\vec{r}} - H(\vec{P},\vec{r})\right]\langle\vec{r}|\psi_t(t)\rangle. \tag{5.5}$$

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With the normalization relation

$$\int \left|\vec{r}\right\rangle \mathrm{d}^3\vec{r} \left\langle \vec{r}\right| = 1, \tag{5.6}$$

we consider the transition matrix element of an arbitrary operator A, from a state $\left|\psi\right\rangle$ with a wavefunction $\left\langle \vec{r}\middle|\psi\right\rangle$, to another state $\left|\phi\right\rangle$ with a wavefunction $\left\langle \vec{r}\middle|\phi\right\rangle$,

$$\left\langle \phi\right| A\left|\psi\right\rangle = \int \underline{\left\langle \phi\middle|\vec{r}\right\rangle \mathrm{d}\vec{r}} \left\langle \vec{r}\right| A\left|\psi\right\rangle = \int \underline{\left\langle \phi\middle|\vec{r}\right\rangle} A\left\langle \vec{r}\middle|\psi\right\rangle \underline{\mathrm{d}\vec{r}}\,. \tag{5.7}$$

From this expression, we obtain the relation between the two representations of this operator, as an operator applied to the state vector $\left|\psi\right\rangle$ and an operator applied to the wavefunction $\left\langle \vec{r}\middle|\psi\right\rangle$,

$$\left\langle \vec{r}\right| A\left|\psi\right\rangle = A\left\langle \vec{r}\middle|\psi\right\rangle. \tag{5.8}$$

With this expression, the dynamic equation (5.5) takes the form

$$\frac{\partial}{\partial t}\left|\psi_t(t)\right\rangle = -\frac{\mathrm{i}}{\hbar}L\left(\vec{P},\vec{r}\right)\left|\psi_t(t)\right\rangle = -\frac{\mathrm{i}}{\hbar}\left[\vec{P}\dot{\vec{r}} - H\left(\vec{P},\vec{r}\right)\right]\left|\psi_t(t)\right\rangle, \tag{5.9}$$

with its conjugated form

$$\frac{\partial}{\partial t}\left\langle \psi_t(t)\right| = \frac{\mathrm{i}}{\hbar}\left\langle \psi_t(t)\right| L\left(\vec{P},\vec{r}\right) = \frac{\mathrm{i}}{\hbar}\left\langle \psi_t(t)\right|\left[\vec{P}\dot{\vec{r}} - H\left(\vec{P},\vec{r}\right)\right]. \tag{5.10}$$

By multiplications of these two equations with the conjugate state vectors,

$$\begin{aligned}
&\frac{\partial}{\partial t}\left|\psi_t(t)\right\rangle\left\langle \psi_t(t)\right| = -\frac{\mathrm{i}}{\hbar}L\left(\vec{P},\vec{r}\right)\left|\psi_t(t)\right\rangle\left\langle \psi_t(t)\right| \\
&\left|\psi_t(t)\right\rangle\frac{\partial}{\partial t}\left\langle \psi_t(t)\right| = \frac{\mathrm{i}}{\hbar}\left|\psi_t(t)\right\rangle\left\langle \psi_t(t)\right| L\left(\vec{P},\vec{r}\right),
\end{aligned} \tag{5.11}$$

and summation of the two equations, we obtain the dynamic equation of the density operator (5.4),

$$\frac{\partial}{\partial t}\rho(t) = -\frac{\mathrm{i}}{\hbar}\left[L\left(\vec{P},\vec{r}\right),\rho(t)\right] = -\frac{\mathrm{i}}{\hbar}\left[\vec{P}\dot{\vec{r}} - H\left(\vec{P},\vec{r}\right),\rho(t)\right]. \tag{5.12}$$

FERMI'S GOLDEN RULE FOR QUANTUM PARTICLE TRANSITIONS IN LAGRANGIAN STATES

We consider the dynamic equation (5.12) for a system with the Hamiltonian $H_0\left(\vec{P},\vec{r}\right)$ and a perturbing potential $V\left(\vec{r}\right)$ with an arbitrary strength ε,

$$\frac{\partial}{\partial t}\rho(t) = -\frac{\mathrm{i}}{\hbar}\left[\vec{P}\dot{\vec{r}} - H_0\left(\vec{P},\vec{r}\right) - \varepsilon V(\vec{r}), \rho(t)\right] = -\frac{\mathrm{i}}{\hbar}\left[L_0\left(\vec{P},\vec{r}\right) - \varepsilon V(\vec{r}), \rho(t)\right], \quad \textbf{(5.13)}$$

which, in the interaction picture,

$$\begin{aligned}\tilde{\rho}(t) &= e^{\frac{\mathrm{i}}{\hbar}L_0\left(\vec{P},\vec{r}\right)t}\rho(t)e^{-\frac{\mathrm{i}}{\hbar}L_0\left(\vec{P},\vec{r}\right)t}\\ \tilde{V}(t) &= e^{\frac{\mathrm{i}}{\hbar}L_0\left(\vec{P},\vec{r}\right)t}Ve^{-\frac{\mathrm{i}}{\hbar}L_0\left(\vec{P},\vec{r}\right)t},\end{aligned} \quad \textbf{(5.14)}$$

takes the simpler form

$$\frac{\partial}{\partial t}\tilde{\rho}(t) = \frac{\mathrm{i}}{\hbar}\left[\varepsilon\tilde{V}(t), \tilde{\rho}(t)\right]. \quad \textbf{(5.15)}$$

Taking into account the effect of this perturbation as a series expansion of the density operator with the strength of this perturbation,

$$\frac{\partial}{\partial t}\left[\tilde{\rho}^{(0)}(t) + \varepsilon\tilde{\rho}^{(1)}(t) + \varepsilon^2\tilde{\rho}^{(2)}(t) + \ldots\right] = \frac{\mathrm{i}}{\hbar}\left[\varepsilon\tilde{V}(t), \tilde{\rho}^{(0)}(t) + \varepsilon\tilde{\rho}^{(1)}(t) + \varepsilon^2\tilde{\rho}^{(2)}(t) + \ldots\right] \quad \textbf{(5.16)}$$

we obtain a system of equations for the terms of this operator generated by this perturbation,

$$\begin{aligned}&\frac{\partial}{\partial t}\tilde{\rho}^{(0)}(t) = 0\\ &\frac{\partial}{\partial t}\tilde{\rho}^{(1)}(t) = \frac{\mathrm{i}}{\hbar}\left[\tilde{V}(t), \tilde{\rho}^{(0)}(t)\right]\\ &\frac{\partial}{\partial t}\tilde{\rho}^{(2)}(t) = \frac{\mathrm{i}}{\hbar}\left[\tilde{V}(t), \tilde{\rho}^{(1)}(t)\right]\\ &\ldots\end{aligned} \quad \textbf{(5.17)}$$

with the solutions

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CHAPTER 6

Dirac's Formalism of General Relativity

TIME-SPACE INTERVAL AND THE METRIC TENSOR

The fundamental law of general relativity, and of physics in general, is the four-dimensionality of the physical space, with the time-space coordinates

$$x = (x^\alpha) = (x^0 = ct, x^1, x^2, x^3) = \left(x^0 = ct, x^i\right), \tag{6.1}$$

satisfying the invariance condition of any time-space interval $\mathrm{d}s$,

$$\mathrm{d}s^2 \equiv c^2 \mathrm{d}\tau^2 = g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta = g_{\alpha'\beta'}\mathrm{d}x^{\alpha'}\mathrm{d}x^{\beta'}, \tag{6.2}$$

for any transformation between two arbitrary systems of coordinates, S, with the metric symmetric tensor $g_{\alpha\beta}$, and S', with the metric symmetric tensor $g_{\alpha'\beta'}$. For the two inverse transformations,

$$\begin{aligned} \mathrm{d}x^{\alpha'} &= \frac{\partial x^{\alpha'}}{\partial x^\alpha}\mathrm{d}x^\alpha \doteq x^{\alpha'}{}_{,\alpha}\mathrm{d}x^\alpha \\ \mathrm{d}x^\alpha &= \frac{\partial x^\alpha}{\partial x^{\alpha'}}\mathrm{d}x^{\alpha'} \doteq x^\alpha{}_{,\alpha'}\mathrm{d}x^{\alpha'} = x^\alpha{}_{,\alpha'}x^{\alpha'}{}_{,\beta}\mathrm{d}x^\beta. \end{aligned} \tag{6.3}$$

we obtain the following relationship:

$$x^\alpha{}_{,\alpha'}x^{\alpha'}{}_{,\beta} = \delta^\alpha_\beta. \tag{6.4}$$

For a flat space, defined by a time-space interval (6.2) of the form

$$\mathrm{d}s^2 = \mathrm{d}x^{0^2} - \mathrm{d}x^{1^2} - \mathrm{d}x^{2^2} - \mathrm{d}x^{3^2} = \mathrm{d}s'^2 = \mathrm{d}x^{0'^2} - \mathrm{d}x^{1'^2} - \mathrm{d}x^{2'^2} - \mathrm{d}x^{3'^2}, \tag{6.5}$$

we consider two systems of coordinates with parallel axes moving to one another in the direction $x^1 \parallel x^{1'}$, as the transformation relations between these coordinates are of the same form:

$$
\begin{aligned}
x^0 &= x^0\left(x^{0'}, x^{1'}\right) \\
x^1 &= x^1\left(x^{0'}, x^{1'}\right) \\
x^2 &= x^{2'} \\
x^3 &= x^{3'}.
\end{aligned} \tag{6.6}
$$

With these expressions, from the equality (6.5) of the two time-space intervals,

$$
\begin{aligned}
\mathrm{d}s^2 &= x^{0^2} - \mathrm{d}x^{1^2} - \mathrm{d}x^{2^2} - \mathrm{d}x^{3^2} \\
&= \left(x^0{}_{,0'}\mathrm{d}x^{0'} + x^0{}_{,1'}\mathrm{d}x^{1'}\right)^2 - \left(x^1{}_{,0'}\mathrm{d}x^{0'} + x^1{}_{,1'}\mathrm{d}x^{1'}\right)^2 - \mathrm{d}x^{2'^2} - \mathrm{d}x^{3'^2} \\
&= \mathrm{d}x^{0'^2} - \mathrm{d}x^{1'^2} - \mathrm{d}x^{2'^2} - \mathrm{d}x^{3'^2}.
\end{aligned}
$$

we obtain the system of the following equations:

$$
\begin{aligned}
&x^0{}_{,0'}{}^2 - x^1{}_{,0'}{}^2 = 1 \\
&x^0{}_{,1'}{}^2 - x^1{}_{,1'}{}^2 = -1 \\
&x^0{}_{,0'}x^0{}_{,1'} - x^1{}_{,0'}x^1{}_{,1'} = 0.
\end{aligned} \tag{6.7}
$$

The transformation of the differentials of the two time-space coordinates in motion,

$$
\begin{aligned}
\mathrm{d}x^0 &= x^0{}_{,0'}\mathrm{d}x^{0'} + x^0{}_{,1'}\mathrm{d}x^{1'} \\
\mathrm{d}x^1 &= x^1{}_{,0'}\mathrm{d}x^{0'} + x^1{}_{,1'}\mathrm{d}x^{1'},
\end{aligned}
$$

with the first two equations (6.7), is

$$
\begin{aligned}
\mathrm{d}x^0 &= \sqrt{1 + x^1{}_{,0'}{}^2}\,\mathrm{d}x^{0'} + x^0{}_{,1'}\mathrm{d}x^{1'} \\
\mathrm{d}x^1 &= x^1{}_{,0'}\mathrm{d}x^{0'} + \sqrt{1 + x^0{}_{,1'}{}^2}\,\mathrm{d}x^{1'}.
\end{aligned} \tag{6.8}
$$

At the same time, with the third equation (6.7),

$$
x^0{}_{,1'} = x^1{}_{,0'}\,\frac{x^1{}_{,1'}}{x^0{}_{,0'}} = x^1{}_{,0'}\,\frac{\sqrt{1 + x^0{}_{,1'}{}^2}}{\sqrt{1 + x^1{}_{,0'}{}^2}}\,.
$$

we obtain the equation

$$\sqrt{\frac{1}{{x^0}_{,1'}^{\ 2}}+1}=\sqrt{\frac{1}{{x^1}_{,0'}^{\ 2}}+1},$$

which reduces to the equality

$$x^0{}_{,1'}=x^1{}_{,0'}. \tag{6.9}$$

With this expression, equations 6.8 become

$$\begin{aligned} dx^0 &= \sqrt{1+{x^1}_{,0'}^{\ 2}}dx^{0'}+x^1{}_{,0'}dx^{1'}=\sqrt{1+{x^1}_{,0'}^{\ 2}}\left(dx^{0'}+\frac{x^1{}_{,0'}}{\sqrt{1+{x^1}_{,0'}^{\ 2}}}dx^{1'}\right)\\ dx^1 &= x^1{}_{,0'}dx^{0'}+\sqrt{1+{x^1}_{,0'}^{\ 2}}dx^{1'}=\sqrt{1+{x^1}_{,0'}^{\ 2}}\left(dx^{1'}+\frac{x^1{}_{,0'}}{\sqrt{1+{x^1}_{,0'}^{\ 2}}}dx^{0'}\right). \end{aligned} \tag{6.10}$$

From these equations, we obtain the velocity of the system S' in the system S as

$$v=\left.\frac{dx^1}{dx^0}\right|_{dx^{1'}=0}=\frac{x^1{}_{,0'}}{\sqrt{1+{x^1}_{,0'}^{\ 2}}}, \tag{6.11}$$

and the coefficient of equation (6.10) is a function of this velocity,

$$\sqrt{1+{x^1}_{,0'}^{\ 2}}=\frac{1}{\sqrt{1-\frac{{x^1}_{,0'}^{\ 2}}{1+{x^1}_{,0'}^{\ 2}}}}=\frac{1}{\sqrt{1-v^2}}. \tag{6.12}$$

With these expressions, we obtain the Lorentz transformation:

$$\begin{aligned} dx^0 &= \frac{dx^{0'}+vdx^{1'}}{\sqrt{1-v^2}}\\ dx^1 &= \frac{dx^{1'}+vdx^{0'}}{\sqrt{1-v^2}}. \end{aligned} \tag{6.13}$$

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CHAPTER 7

Quantum Particles in Gravitational and Electromagnetic Fields

GRAVITATIONAL AND ELECTROMAGNETIC FIELDS' ACTIONS

We describe the interaction of a quantum particle with an electromagnetic field in the curved space of a gravitational field, by the invariant action integrals with the gravitational field,

$$I_g = \int R\sqrt{-g}\mathrm{d}^4x\,, \qquad \textbf{(7.1)}$$

mass,

$$I_m = -\int \rho\sqrt{-g}\mathrm{d}^4x\,, \qquad \textbf{(7.2)}$$

electromagnetic field,

$$I = \int F_{\mu\nu}F^{\nu\mu}\sqrt{-g}\mathrm{d}^4x\,, \qquad \textbf{(7.3)}$$

the function of a potential A_μ,

$$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} = A_{\mu:\nu} - A_{\nu:\mu}\,, \qquad \textbf{(7.4)}$$

electric charge

$$I_q = -\int A_\mu \tilde{j}^\mu \sqrt{-g}\mathrm{d}^4x\,, \qquad \textbf{(7.5)}$$

the function of the charge flow density,

$$\tilde{j}^\mu = \tilde{\rho} v^\mu\,, \qquad \textbf{(7.6)}$$

and the electromagnetic potential A_μ.

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GRAVITATIONAL ACTION INTEGRAL

For the gravitational field action integral (7.1), from the expression (6.86) of the curvature tensor,

$$R^{\alpha}_{\mu\nu\sigma} = -\Gamma^{\alpha}_{\mu\nu,\sigma} + \Gamma^{\alpha}_{\mu\sigma,\nu} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\sigma} + \Gamma^{\beta}_{\mu\sigma}\Gamma^{\alpha}_{\beta\nu} ,$$

by the contraction $\sigma = \alpha$, we obtain the Ricci tensor

$$R_{\mu\nu} = R_{\nu\mu} = R^{\alpha}_{\mu\nu\alpha} = -\Gamma^{\alpha}_{\mu\nu,\alpha} + \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha} + \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu} , \tag{7.7}$$

with the symmetry of the second term obtained from expression (6.74),

$$\Gamma^{\alpha}_{\mu\alpha,\nu} = \left(\ln\sqrt{-g}\right)_{,\mu\nu} .$$

With the Ricci tensor (7.7), we obtain the total curvature as the difference of two terms,

$$R = g^{\mu\nu}R_{\mu\nu} = R^{*} - L , \tag{7.8}$$

i.e., the first-order term,

$$R^{*} = g^{\mu\nu}\left(\Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha}\right), \tag{7.9}$$

and the second-order term,

$$L = g^{\mu\nu}\left(\Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha} - \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}\right). \tag{7.10}$$

In the gravitation action integral (7.1), by the partial integration, we eliminate the differentials of the Christoffel symbols in the term (7.9),

$$\begin{aligned} R^{*}\sqrt{-g} &= g^{\mu\nu}\Gamma^{\alpha}_{\mu\alpha,\nu}\sqrt{-g} - g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu,\alpha}\sqrt{-g} \\ &= \left(g^{\mu\nu}\Gamma^{\alpha}_{\mu\alpha}\sqrt{-g}\right)_{,\nu} - \Gamma^{\alpha}_{\mu\alpha}\left(g^{\mu\nu}\sqrt{-g}\right)_{,\nu} - \left(g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}\sqrt{-g}\right)_{,\alpha} + \Gamma^{\alpha}_{\mu\nu}\left(g^{\mu\nu}\sqrt{-g}\right)_{,\alpha} , \end{aligned} \tag{7.11}$$

leading to the reduction of this term into an expression depending only on derivatives of the metric tensor,

$$R^*\sqrt{-g}=\Gamma^{\alpha}_{\mu\nu}\left(g^{\mu\nu}\sqrt{-g}\right)_{,\alpha}-\Gamma^{\alpha}_{\mu\alpha}\left(g^{\mu\nu}\sqrt{-g}\right)_{,\nu}. \quad \textbf{(7.12)}$$

With the expressions (6.74),

$$\left(\sqrt{-g}\right)_{,\nu}=\Gamma^{\mu}_{\nu\mu}\sqrt{-g}, \quad \textbf{(7.13)}$$

and (6.50), we also eliminate the differentials of the metric tensor in the first term,

$$\begin{aligned}\left(g^{\mu\nu}\sqrt{-g}\right)_{,\alpha}&=g^{\mu\nu}{}_{,\alpha}\sqrt{-g}+g^{\mu\nu}\sqrt{-g}_{,\alpha}=-g^{\mu\rho}g^{\nu\sigma}g_{\rho\sigma,\alpha}\sqrt{-g}+g^{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma}\sqrt{-g}\\&=-g^{\mu\rho}g^{\nu\sigma}\left(\Gamma_{\rho\sigma\alpha}+\Gamma_{\sigma\rho\alpha}\right)\sqrt{-g}+g^{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma}\sqrt{-g}\\&=\left(-g^{\nu\sigma}\Gamma^{\mu}_{\sigma\alpha}-g^{\mu\rho}\Gamma^{\nu}_{\rho\alpha}+g^{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma}\right)\sqrt{-g},\end{aligned} \quad \textbf{(7.14)}$$

and by an index contraction, also in the second term,

$$\left(g^{\mu\nu}\sqrt{-g}\right)_{,\nu}=\left(-g^{\nu\sigma}\Gamma^{\mu}_{\sigma\nu}-\underline{\cancel{g^{\mu\rho}\Gamma^{\nu}_{\rho\nu}}}+\underline{\cancel{g^{\mu\nu}\Gamma^{\sigma}_{\nu\sigma}}}\right)\sqrt{-g}=-g^{\nu\sigma}\Gamma^{\mu}_{\sigma\nu}\sqrt{-g}. \quad \textbf{(7.15)}$$

With these terms, from expressions (7.12) and (7.10), we obtain

$$\begin{aligned}R^*&=\left(\underline{-g^{\nu\sigma}\Gamma^{\mu}_{\sigma\alpha}-g^{\mu\rho}\Gamma^{\nu}_{\rho\alpha}}+\underline{\underline{g^{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma}}}\right)\Gamma^{\alpha}_{\mu\nu}+\underline{\underline{g^{\nu\sigma}\Gamma^{\mu}_{\sigma\nu}\Gamma^{\alpha}_{\mu\alpha}}}\\&=2\left(g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}\Gamma^{\sigma}_{\alpha\sigma}-g^{\mu\sigma}\Gamma^{\alpha}_{\mu\nu}\Gamma^{\nu}_{\sigma\alpha}\right)\\&=2g^{\mu\nu}\left(\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}-\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu}\right)=2L,\end{aligned} \quad \textbf{(7.16)}$$

as the total curvature (7.8) is

$$R=R^*-L=2L-L=L. \quad \textbf{(7.17)}$$

With this expression, the gravitational action (7.1) is of the form of a time-integral

$$I_g=\int\mathcal{L}\mathrm{d}^4x=\int\mathrm{d}x^0\int\mathcal{L}\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3, \quad \textbf{(7.18)}$$

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CHAPTER 8

Black Hole Matter Dynamics

PARTICLE DYNAMICS IN A BLACK HOLE

In equations (6.166) and (6.167), we remark the two singularities, $r=0$, and the Schwarzschild radius $r=r_0=2m$ as the integration constant (6.146) of the differential equation (6.140), given by the mass of the matter inside the boundary determined by this radius, which is the source of the gravitational potential (6.148). We consider a radial motion

$$\frac{\mathrm{d}x^2}{\mathrm{d}s}=\frac{\mathrm{d}x^3}{\mathrm{d}s}=0. \tag{8.1}$$

According to the geodesic equation (6.83) for the time coordinate acceleration in the time-space interval,

$$\begin{aligned}\frac{\mathrm{d}^2x^0}{\mathrm{d}s^2}&=-\Gamma^0_{01}\frac{\mathrm{d}x^0}{\mathrm{d}s}\frac{\mathrm{d}x^1}{\mathrm{d}s}-\Gamma^0_{10}\frac{\mathrm{d}x^1}{\mathrm{d}s}\frac{\mathrm{d}x^0}{\mathrm{d}s}=-2g^{00}\Gamma_{001}\frac{\mathrm{d}x^0}{\mathrm{d}s}\frac{\mathrm{d}x^1}{\mathrm{d}s}\\&=-g^{00}\left(g_{00,1}+g_{01,0}-g_{01,0}\right)\frac{\mathrm{d}x^0}{\mathrm{d}s}\frac{\mathrm{d}x^1}{\mathrm{d}s}=-g^{00}g_{00,1}\frac{\mathrm{d}x^0}{\mathrm{d}s}\frac{\mathrm{d}x^1}{\mathrm{d}s}\\&=-g^{00}\frac{\mathrm{d}g_{00}}{\mathrm{d}s}\frac{\mathrm{d}x^0}{\mathrm{d}s}.\end{aligned} \tag{8.2}$$

With the relation (6.166) between the contravariant and covariant metric elements,

$$g^{00}=\frac{1}{g_{00}}, \tag{8.3}$$

we obtain the equation

$$g_{00}\frac{\mathrm{d}^2x^0}{\mathrm{d}s^2}+\frac{\mathrm{d}g_{00}}{\mathrm{d}s}\frac{\mathrm{d}x^0}{\mathrm{d}s}=\frac{\mathrm{d}}{\mathrm{d}s}\left(g_{00}\frac{\mathrm{d}x^0}{\mathrm{d}s}\right)=0, \tag{8.4}$$

for the time coordinate velocity

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$$v^0 = \frac{\mathrm{d}x^0}{\mathrm{d}s}. \tag{8.5}$$

By integration, we obtain the equation

$$g_{00} v^0 = k_0 , \tag{8.6}$$

with the integration constant k_0. With the fundamental equation (2.31),

$$g_{00} {v^0}^2 + g_{11} {v^1}^2 = 1 , \tag{8.7}$$

for a particle attracted in the central gravitational field considered here, we obtain the radial velocity in the proper time,

$$v^1 = -\left(\frac{1 - k_0 v^0}{g_{11}} \right)^{1/2} < 0 . \tag{8.8}$$

With the relation obtained from the Schwarzschild expression (6.166) for the two metric elements considered here,

$$g_{00} = -\frac{1}{g_{11}} = 1 - \frac{2m}{r} , \tag{8.9}$$

and equation (8.6), we obtain the radial velocity

$$v^1 = -\left({k_0}^2 - g_{00} \right)^{1/2} = -\left({k_0}^2 + \frac{2m}{r} - 1 \right)^{1/2} , \tag{8.10}$$

as the velocity of the time coordinate in the proper time is

$$v^0 = \frac{k_0}{1 - \frac{2m}{r}} . \tag{8.11}$$

From these expressions, we obtain the particle velocity in the local time,

$$\frac{\mathrm{d}x^1}{\mathrm{d}x^0}=\frac{v^1}{v^0}=-\frac{1}{k_0}\left(k_0^{\ 2}+\frac{2m}{r}-1\right)^{1/2}\left(1-\frac{2m}{r}\right). \quad \textbf{(8.12)}$$

With the integration constant $k_0=1$, for satisfying the boundary condition

$$\lim_{r\to\infty}\frac{\mathrm{d}x^1}{\mathrm{d}x^0}=0. \quad \textbf{(8.13)}$$

The radial velocity (8.12) takes the explicit form

$$\frac{\mathrm{d}r}{\mathrm{d}x^0}=-\left(1-\frac{r_0}{r}\right)\left(\frac{r_0}{r}\right)^{1/2}, \quad \textbf{(8.14)}$$

depending on the Schwarzschild radius $r_0=2m$, which, according to (6.146), is

$$r_0=2m=2\frac{G}{c^2}M_G=1.4849\times10^{-27}\left[\ \mathrm{m\,Kg^{-1}}\right]M_G\,. \quad \textbf{(8.15)}$$

In a gravitational field with spherical symmetry of a body with the mass M_G, we obtain a particle acceleration

$$\begin{aligned}\frac{\mathrm{d}^2r}{\mathrm{d}x^{0^2}}&=-\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(1-\frac{r_0}{r}\right)\left(\frac{r_0}{r}\right)^{1/2}\right]\frac{\mathrm{d}r}{\mathrm{d}x^0}\\
&=\left[\frac{r_0}{r^2}\left(\frac{r_0}{r}\right)^{1/2}+\left(1-\frac{r_0}{r}\right)\frac{1}{2}\left(\frac{r_0}{r}\right)^{-1/2}\left(-\frac{r_0}{r^2}\right)\right]\left(1-\frac{r_0}{r}\right)\left(\frac{r_0}{r}\right)^{1/2}\\
&=\left[\frac{r_0}{r^2}\frac{r_0}{r}+\left(1-\frac{r_0}{r}\right)\frac{1}{2}\left(-\frac{r_0}{r^2}\right)\right]\left(1-\frac{r_0}{r}\right)\\
&=-\left[-\frac{r_0}{r}+\left(1-\frac{r_0}{r}\right)\frac{1}{2}\right]\left(1-\frac{r_0}{r}\right)\frac{r_0}{r^2}.\end{aligned} \quad \textbf{(8.16)}$$

For a smaller mass M_G, as the mass of our planet, $M_G=5.972\times10^{24}\ \mathrm{Kg}$, with a Schwarzschild radius (8.15), $r_0=1.4849\times10^{-27}\times5.972\times10^{24}=8.868\times10^{-3}\ \mathrm{m}$,

CHAPTER 9

Our Universe as a System of Visible Bodies

Our previous results, namely, the beginning of a black hole with an explosion of the central matter, a bang followed by a continuous displacement of the inside matter towards the Schwarzschild boundary, but reaching this boundary only in an infinite time, suggest that our universe, characterized by Big Bang, inflation, and a small large-scale density, is, in fact, a large black hole in the total everlasting universe. From the radius of our universe as a black hole, $r_0 = 2m$, evaluated as $4.4\times10^{26}\,\mathrm{m}$ [37], significantly larger than the radius of the observable universe, $1.3055904\times10^{26}\mathrm{m}$, we obtain the total mass of this universe, $M_G = \frac{c^2}{G}m > 3.0\times10^{53}\,\mathrm{Kg}$, mainly situated in the neighborhood of the Schwarzschild boundary, which is somehow in agreement with the empirical evaluation of the mass of the observable universe, $1.5\times10^{53}\,\mathrm{Kg}$ [37]. Of course, the gravitational motion described by equations (8.34) - (8.50) is strongly perturbed by the electromagnetic and nuclear interactions, strongly excited by the explosion of the central matter described by the velocity (8.18), colliding the matter around, and spreading this matter throughout the whole universe. This matter, mainly traveling towards the Schwarzschild boundary of the universe but reaching this boundary only in an infinite time, is slightly decelerated in the total gravitational field of this universe, according to equation (8.19) for a rather large radius,

$$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} < -\left[3\left(\frac{2m}{r}-1\right)+2\right]\left(\frac{2m}{r}-1\right)\frac{mc^2}{r^2}\,. \quad \textbf{(9.1)}$$

For a particle with a radius $r = r_0/2 = m$, we obtain an acceleration approximately 10 orders of magnitude smaller than the acceleration at the surface of Earth,

$$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} < -5\frac{c^2}{m} = -5\frac{9\times10^{16}\,\mathrm{m^2s^{-2}}}{2.2\times10^{26}\,\mathrm{m}} = -2.45\times10^{-9}\,\mathrm{m\,s^{-2}}\,. \quad \textbf{(9.2)}$$

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Of course, this acceleration is about 6 orders of magnitude smaller than the Earth's acceleration a_E around the Sun on its trajectory with the radius $r_E = 150\times 10^9\,\mathrm{m}$, which, with the velocity

$$v_E = \frac{2\pi r_E}{1\,\mathrm{y}} = \frac{2\pi\times 150\times 10^9\,\mathrm{m}}{365\,\mathrm{d}\ \times 24\ \mathrm{h}\ \times 60\ \mathrm{min}\ \times 60\ \mathrm{s}} = 3\times 10^4\ \mathrm{m\ s^{-1}}, \quad \textbf{(9.3)}$$

is

$$a_E = \frac{v_E{}^2}{r_E} = \frac{9\times 10^8\,\mathrm{m^2 s^{-2}}}{150\times 10^9\,\mathrm{m}} = 6\times 10^{-3}\ \mathrm{m\ s^{-2}}, \quad \textbf{(9.4)}$$

which is strongly perturbed by the large systems of celestial bodies traveling in all directions through the universe. These bodies, with external interactions much stronger than the gravitational interactions, spreading throughout the whole universe, tend to cancel the universal acceleration (9.2). When the Big Bang-Inflation process is finished, the inside parts of the large black hole of our universe, formed of celestial bodies as are known now, begin to concentrate gravitationally, the initial energy of the Big Bang being dissipated by the strong electromagnetic and nuclear interactions. Evidently, the visible universe, corresponding to the age of our universe with the light velocity, is much smaller than the internal universe corresponding to this age with the much larger velocity of the Big Bang-Inflation process with the radius $r_{0I} < r_0$. An observable quantity is a redshift, which is a decrease in the frequency of the light with frequency ω_0 emitted by an atom at the radius r_2 in the visible universe. The smaller frequency ω_1 at a smaller radius r_1, where this frequency is measured, can be understood from the Schwarzschild metric tensor (6.166) with the local time variation (8.44), with a Schwarzschild radius r_{0I} for the gravitational field of the internal universe,

$$\Delta s=\sqrt{g_{00}\Delta x_1^{0^2}+g_{11}\Delta x_1^{1^2}}=\frac{2\pi c}{\omega_1}\sqrt{1-\frac{r_{0I}}{r_1}-\left(1-\frac{r_{0I}}{r_1}\right)^{-1}\left(\frac{\Delta r_1}{\Delta x^0}\right)^2}$$
$$=\frac{2\pi c}{\omega_1}\sqrt{1-\frac{r_{0I}}{r_1}-\left(1-\frac{r_{0I}}{r_1}\right)^{-1}\left(\frac{r_{0I}}{r_1}-1\right)^2\frac{r_{0I}}{r_1}}=\frac{2\pi c}{\omega}\sqrt{\left(1-\frac{r_{0I}}{r_1}\right)\left(1-\frac{r_{0I}}{r_1}\right)} \quad \textbf{(9.5)}$$
$$=\frac{2\pi c}{\omega_1}\left(1-\frac{r_{0I}}{r_1}\right)=\frac{2\pi c}{\omega_2}\left(1-\frac{r_{0I}}{r_2}\right)=\frac{2\pi c}{\omega_0}, \qquad r_2>>r_1>r_{0I}$$

which means a redshift $\Delta\omega_{01}=\omega_0-\omega_1=\omega_0-\omega_1+\omega_2-\omega_2=\omega_0-\omega_2+\Delta\omega_{21}$ of the light emitted at the distance r_2-r_1,

$$\Delta\omega_{21}=\omega_2-\omega_1=\frac{2\pi c}{\Delta s}\left(1-\frac{r_{0I}}{r_2}\right)-\frac{2\pi c}{\Delta s}\left(1-\frac{r_{0I}}{r_1}\right)=\frac{2\pi c}{\Delta s}\left(\frac{r_{0I}}{r_1}-\frac{r_{0I}}{r_2}\right)=\frac{2\pi c r_{0I}}{\Delta s}\frac{r_2-r_1}{r_1 r_2}. \quad \textbf{(9.6)}$$

The redshift variation with the distance of the emitting atom r_2-r_1, $\frac{\mathrm{d}\Delta\omega_{01}}{-\mathrm{d}r_1}=-\frac{\mathrm{d}\Delta\omega_{21}}{\mathrm{d}r_1}=\frac{2\pi c r_{0I}}{\Delta s\cdot r_1^2}$, as a gravitational effect, is much larger than the redshift variation in a linear approximation $\frac{\mathrm{d}\Delta\omega_{21}}{\mathrm{d}r_2}=\frac{2\pi c r_{0I}}{\Delta s\cdot r_1 r_2}$, which is in agreement with a phenomenon considered in conventional cosmology as an effect of a dark energy, which would increase the expansion of our universe at far distances r_2, according to the Doppler effect. Here, we have taken into account that the velocity of our galaxy, much closer to the center of the universe, is much larger than the velocity of a far galaxy, approaching this center with a much smaller velocity.

We consider a quantum particle in the proper time τ,

$$\psi\left(x^i,\tau\right)=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\varphi\left(p^j,\tau\right)e^{\frac{\mathrm{i}}{\hbar}\left[p^j x^j-L\left(x^\alpha,v^\alpha\right)\tau\right]}\mathrm{d}^3p$$
$$\varphi\left(p^j,\tau\right)=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\psi\left(x^i,\tau\right)e^{-\frac{\mathrm{i}}{\hbar}\left[p^j x^j-L\left(x^\alpha,v^\alpha\right)\tau\right]}\mathrm{d}^3x, \quad \textbf{(9.7)}$$

with the Lagrangian

CHAPTER 10

Gravitational Wave, Graviton Spin, and Particle Spin

GRAVITATIONAL WAVE

Following Dirac [24], we consider a gravitational wave in the vacuum as a system of harmonic coordinates, which, by definition, are scalars satisfying the covariance of the d'Alembert equation,

$$\square_{:} x^{\lambda} = 0 , \quad \textbf{(10.1)}$$

with the explicit form

$$g^{\mu\nu} x^{\lambda}{}_{:\mu\nu} = g^{\mu\nu} \left(x^{\lambda}{}_{,\mu} \right)_{:\nu} = g^{\mu\nu} \left(x^{\lambda}{}_{,\mu\nu} - \Gamma^{\alpha}_{\mu\nu} x^{\lambda}{}_{,\alpha} \right) = 0 . \quad \textbf{(10.2)}$$

Since $x^{\lambda}{}_{,\mu\nu} = \left(\delta^{\lambda}_{\mu} \right)_{,\nu} = 0$, we obtain the coordinate variations as functions of the gravitational curvature, as described by the Christoffel symbols,

$$g^{\mu\nu} \Gamma^{\alpha}_{\mu\nu} = 0 , \quad \textbf{(10.3)}$$

or

$$g^{\mu\nu} \Gamma_{\alpha\mu\nu} = g^{\mu\nu} \frac{1}{2} \left(g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha} \right) = 0 , \quad \textbf{(10.4)}$$

which is

$$g^{\mu\nu} \left(g_{\alpha\mu,\nu} - \frac{1}{2} g_{\mu\nu,\alpha} \right) = 0 . \quad \textbf{(10.5)}$$

By differentiating this equation with x^{β} and neglecting the second-order terms, we obtain a second-order differential equation as:

$$g^{\mu\nu}\left(\underline{\underline{g_{\alpha\mu,\nu\beta}}}-\frac{1}{2}\underline{g_{\mu\nu,\alpha\beta}}\right)=0\,, \qquad \textbf{(10.6)}$$

and, by the interchange of the indices α and β, the similar equation can be written as:

$$g^{\mu\nu}\left(\underline{\underline{\underline{g_{\beta\mu,\nu\alpha}}}}-\frac{1}{2}\underline{g_{\mu\nu,\alpha\beta}}\right)=0\,. \qquad \textbf{(10.7)}$$

From the sum of these equations, with the null Ricci tensor (6.136), a gravitational field propagating in the vacuum can be described, which, with these notations, is calculated as:

$$2R_{\alpha\beta}=g^{\mu\nu}\left(\underline{g_{\mu\nu,\alpha\beta}}-\underline{\underline{\underline{g_{\mu\beta,\alpha\nu}}}}+g_{\alpha\beta,\mu\nu}-\underline{\underline{g_{\alpha\nu,\mu\beta}}}\right)=0\,. \qquad \textbf{(10.8)}$$

For the description of the gravitational waves, we obtain the d'Alembert equation with total derivative for the metric tensor, which is the gravitational potential, similar to the d'Alembert equation (10.1) - (10.2) with covariant derivatives in harmonic coordinates,

$$g^{\mu\nu}g_{\alpha\beta,\mu\nu}=0\,. \qquad \textbf{(10.9)}$$

QUANTUM PARTICLES IN A GRAVITATIONAL WAVE

We consider a quantum particle in a gravitational wave as:

$$\begin{aligned}\psi\left(x^i,t\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\varphi\left(P^j,t\right)e^{\frac{\mathrm{i}}{\hbar}\left[P^jx^j-L\left(x^\alpha,\dot{x}^\alpha\right)t\right]}\mathrm{d}^3P\\ \varphi\left(P^j,t\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\psi\left(x^i,t\right)e^{-\frac{\mathrm{i}}{\hbar}\left[P^jx^j-L\left(x^\alpha,\dot{x}^\alpha\right)t\right]}\mathrm{d}^3x\,,\end{aligned} \qquad \textbf{(10.10)}$$

with the Lagrangian function as:

$$L\left(x^\alpha,\dot{x}^\alpha\right)=-Mc^2\sqrt{g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}\,. \qquad \textbf{(10.11)}$$

For simplicity, by taking into account that such waves could be generated only by heavy celestial bodies, we consider only slow velocities,

$$\dot{x}^i << \dot{x}^0 \simeq 1. \tag{10.12}$$

This means that the velocities in the local time t are approximately equal to the velocities in the proper time τ, $\dot{x}^\alpha = v^\alpha$, as the expression (10.11) is the square root of the fundamental invariant (2.31),

$$I_0 = g_{\alpha\beta} v^\alpha v^\beta \simeq g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 1. \tag{10.13}$$

From (10.11), with (10.13) and a time-space diagonalization for the momentum in the local time, we obtain the simpler expression as:

$$\begin{aligned} p^j &= \frac{\partial L}{c\partial \dot{x}^j} = -Mc^2 \frac{\partial}{c\partial \dot{x}^j} \sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta} = -Mc \frac{1}{2\sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}} \frac{\partial}{\partial \dot{x}^j}\left(g_{ji}\dot{x}^j \dot{x}^i + g_{ij}\dot{x}^i \dot{x}^j \right) \\ &= -Mc \frac{g_{ji}\dot{x}^i + g_{ij}\dot{x}^i}{2\sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}} = -Mc g_{ij}\dot{x}^i. \end{aligned} \tag{10.14}$$

The force acting on a quantum particle in the gravitational field, as a time derivative of the momentum (10.14), is also obtained as the wave/group velocity of the wavefunction (10.10) in the momentum space, with the Lagrangian (10.11) and the approximate relation (10.13),

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} p^j &= c\dot{p}^j = -\underline{Mc^2 g_{ij,k}\dot{x}^k \dot{x}^i} - Mc^2 g_{ij}\ddot{x}^i = \frac{\partial L}{\partial x^j} \\ &= -Mc^2 \frac{\dot{x}^\alpha \dot{x}^\beta g_{\alpha\beta,j}}{2\sqrt{g_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta}} = -\frac{1}{2}\underline{Mc^2 g_{\alpha\beta,j}\dot{x}^\alpha \dot{x}^\beta}. \end{aligned}$$

Multiplying this expression with g^{kj} and taking into account the inequality (10.12), we obtain the gravitational acceleration proportional to the gradient of the metric tensor element g_{00} as a gravitational potential as follows:

$$\ddot{x}^k = \frac{1}{2} g^{kj} g_{00,j}. \tag{10.15}$$

CHAPTER 11

Applications to Quantum Electrodynamics

ELECTROMAGNETIC INTERACTION BY PHOTON EXCHANGE

In agreement with quantum electrodynamics [34], the electromagnetic interaction between two charged quantum particles can be conceived by an exchange of photons, as described in Chapter 3. The dynamics of a particle, in the proper time of the collisional time-space domain, can be described by the dynamic equation (2.54) with a null velocity, $\dot{\vec{r}} = 0$, as a Schrödinger-like equation, with a Hamiltonian H_0 for a basis of eigenstates ϕ_k,

$$H_0\phi_k = E_k\phi_k, \qquad \langle \phi_j | \phi_k \rangle = \delta_{jk}, \tag{11.1}$$

and an interaction potential $V(\vec{r},t)$,

$$-\mathrm{i}\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = \left[H_0 + V(\vec{r},t) \right]\psi(\vec{r},t), \tag{11.2}$$

with a solution

$$\psi(\vec{r},t) = \sum_k c_k(t)\phi_k(\vec{r}) e^{\frac{\mathrm{i}}{\hbar}E_k t}. \tag{11.3}$$

Depending on the dynamic amplitudes $c_k(t)$, from (11.2), we obtain the dynamic equation as:

$$-\mathrm{i}\sum_k \left[\hbar \frac{\mathrm{d}c_k}{\mathrm{d}t}\phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t} + \underline{\mathrm{i}E_k c_k \phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t}} \right] = \sum_k \underline{H_0 c_k \phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t}} + \sum_k V c_k \phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t}, \tag{11.4}$$

which, with the eigenstate equation (11.1), is

$$\sum_k \frac{\mathrm{d}c_k}{\mathrm{d}t}\phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t} = \frac{\mathrm{i}}{\hbar}\sum_k c_k(t) V \phi_k e^{\frac{\mathrm{i}}{\hbar}E_k t}, \tag{11.5}$$

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For the final state $|f\rangle$, from the scalar product $\langle f|k\rangle$, we obtain the time-variation of the corresponding dynamic amplitude,

$$\frac{\mathrm{d}c_f}{\mathrm{d}t} = \frac{\mathrm{i}}{\hbar}\sum_k c_k(t)\langle f|V|k\rangle e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_k)t}, \tag{11.6}$$

as a sum of transitions $|k\rangle \to |f\rangle$ determined by the transition matrix elements of the interaction potential, $\langle f|V|k\rangle$, with the dynamic amplitudes $c_k(t)$. These amplitudes can be readily obtained from equation (11.5) in the first-order approximation for the initial state $|i\rangle$,

$$\sum_{k'} \frac{\mathrm{d}c_{k'}}{\mathrm{d}t}\phi_{k'}e^{\frac{\mathrm{i}}{\hbar}E_{k'}t} = \frac{\mathrm{i}}{\hbar}\sum_{k'} Vc_{k'}(t)\phi_{k'}e^{\frac{\mathrm{i}}{\hbar}E_{k'}t} \simeq \frac{\mathrm{i}}{\hbar}V\phi_i e^{\frac{\mathrm{i}}{\hbar}E_i t}.$$

By the scalar product with the state $|k\rangle$ and time integration, we obtain the dynamic amplitudes

$$c_k(t) = \frac{\mathrm{i}}{\hbar}\int_0^t \langle k|V|i\rangle e^{-\frac{\mathrm{i}}{\hbar}(E_k - E_i)t'}\mathrm{d}t' = \mathrm{i}\langle k|V|i\rangle \frac{e^{-\frac{\mathrm{i}}{\hbar}(E_k - E_i)t}}{E_i - E_k}, \tag{11.7}$$

as determined by the transition matrix elements of the interaction potential $\langle k|V|i\rangle$. With this expression, the time-variation of the final state dynamic amplitude (11.6) takes the form as

$$\frac{\mathrm{d}c_f}{\mathrm{d}t} = -\frac{1}{\hbar}\sum_k \frac{\langle f|V|k\rangle\langle k|V|i\rangle}{E_i - E_k} e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_i)t} = -\frac{1}{\hbar}T_{fi}e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_i)t}, \tag{11.8}$$

depending on the transition matrix element

$$T_{fi} = \sum_k \frac{\langle f|V|k\rangle\langle k|V|i\rangle}{E_i - E_k}. \tag{11.9}$$

From (11.8), we obtain the dynamic amplitude of the final state $|f\rangle$ reached in the time T,

$$c_f(T) = -\frac{1}{\hbar}\int_0^T T_{fi} e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_i)t}\mathrm{d}t = -\frac{T_{fi}}{\hbar}\int_0^T e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_i)t}\mathrm{d}t, \tag{11.10}$$

which means a transition probability

$$P_{fi} = c_f^*(T)c_f(T) = \frac{1}{\hbar^2}\left|T_{fi}\right|^2 \int_0^T\int_0^T e^{\frac{\mathrm{i}}{\hbar}(E_f - E_i)t} e^{-\frac{\mathrm{i}}{\hbar}(E_f - E_i)t'}\mathrm{d}t\mathrm{d}t'. \tag{11.11}$$

With a time origin at the middle of the transition time interval T and the transition frequency

$$\omega = \frac{E_f - E_i}{\hbar}, \tag{11.12}$$

we obtain the transition rate in a final state $|f\rangle$,

$$\begin{aligned}\Gamma_{fi} &= \frac{1}{\hbar^2}\left|T_{fi}\right|^2 \lim_{T\to\infty}\left(\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{\mathrm{i}(E_f - E_i)t} e^{-\mathrm{i}(E_f - E_i)t'}\mathrm{d}t\mathrm{d}t'\right) \\ &= \frac{2\pi}{\hbar^2}\left|T_{fi}\right|^2 \lim_{T\to\infty}\left(\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{\mathrm{i}(E_f - E_i)t}\delta(E_f - E_i)\mathrm{d}t\right)\end{aligned} \tag{11.13}$$

With this expression, we obtain the transition rate to all the accessible final states as a function of the density of the n final state as

$$\begin{aligned}\Gamma_{\{f\}i} &= \frac{2\pi}{\hbar^2}\int\left|T_{fi}\right|^2 \frac{\mathrm{d}n}{\mathrm{d}E_f}\lim_{T\to\infty}\left(\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{\mathrm{i}(E_f - E_i)t}\delta(E_f - E_i)\mathrm{d}t\right)\mathrm{d}E_f \\ &= \frac{2\pi}{\hbar^2}\int\left|T_{fi}\right|^2 \frac{\mathrm{d}n}{\mathrm{d}E_f}\delta(E_f - E_i)\lim_{T\to\infty}\left(\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\mathrm{d}t\right)\mathrm{d}E_f \\ &= \frac{2\pi}{\hbar^2}\left|T_{fi}\right|^2 \left|\frac{\mathrm{d}n}{\mathrm{d}E_f}\right|_{E_f - E_i},\end{aligned} \tag{11.14}$$

which is the Fermi's golden rule with the transition matrix element (11.9). This matrix element is considered for a transition $|i\rangle \to |f\rangle$ with an intermediate state $|k\rangle$

CHAPTER 12

Grand Unified Theory – Gravitational Electro-magnetic- Flavor- and Chromo Dynamics as a Single Theory

GRAVITATIONAL, ELECTROMAGNETIC, WEAK, AND STRONG FIELDS

In chapter 2, we describe a black quantum particle in the two conjugate spaces of the coordinates $\left\{x^i\right\}$ and momentum $\left\{p^j\right\}$, by the distribution of matter amplitudes (2.34),

$$\begin{aligned}\psi\left(x^j,\tau\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\varphi\left(p^j,\tau\right)e^{\frac{i}{\hbar}\left[p^jx^j+Mc^2\sqrt{g_{\alpha\beta}v^\alpha v^\beta}\tau\right]}\mathrm{d}^3p\\ \varphi\left(p^j,\tau\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\psi\left(x^j,\tau\right)e^{-\frac{i}{\hbar}\left[p^jx^j+Mc^2\sqrt{g_{\alpha\beta}v^\alpha v^\beta}\tau\right]}\mathrm{d}^3x,\end{aligned} \tag{12.1}$$

in the proper time τ, depending on the general relativistic Lagrangian (2.32),

$$L\left(x^\alpha,v^\alpha\right)=-Mc^2\sqrt{g_{\alpha\beta}v^\alpha v^\beta}\,, \tag{12.2}$$

which, in the metric tensor $g_{\alpha\beta}$, includes the gravitational field. However, since the electromagnetic field is much stronger than the gravitational one, in chapter 3, we describe a particle with an electric charge e in the approximation of the special relativity by wavefunctions of the same form

$$\begin{aligned}\psi\left(\vec{r},t\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\varphi\left(\vec{P},t\right)e^{\frac{i}{\hbar}\left[\vec{P}\vec{r}-L\left(\vec{r},\dot{\vec{r}},t\right)t\right]}\mathrm{d}^3\vec{P}\\ \varphi\left(\vec{P},t\right)&=\frac{1}{\left(2\pi\hbar\right)^{3/2}}\int\psi\left(\vec{r},t\right)e^{-\frac{i}{\hbar}\left[\vec{P}\vec{r}-L\left(\vec{r},\dot{\vec{r}},t\right)t\right]}\mathrm{d}^3\vec{r},\end{aligned} \tag{12.3}$$

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with the particle field Lagrangian, including an electromagnetic field with the potentials $U(\vec{r})$, $\vec{A}(\vec{r},t)$,

$$L\left(\vec{r},\dot{\vec{r}},t\right)=-Mc^2\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}-eU(\vec{r})+e\vec{A}(\vec{r},t)\dot{\vec{r}}\ , \tag{12.4}$$

and the momentum

$$\vec{P}=\frac{\partial}{\partial\dot{\vec{r}}}L\left(\vec{r},\dot{\vec{r}},t\right)=\vec{p}+e\vec{A}(\vec{r},t)\,, \tag{12.5}$$

as the sum of the mechanical momentum

$$\vec{p}=\frac{M\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}}\ , \tag{12.6}$$

with the electromagnetic momentum $e\vec{A}(\vec{r},t)$. From the conservation of the particle-field momentum $\vec{P}$, and energy E, with the canonical Hamiltonian,

$$\begin{aligned} H\left(\vec{P},\vec{r}\right) &\doteq c\sqrt{M^2c^2+\vec{p}^2}+eU(\vec{r})=c\sqrt{M^2c^2+\left[\vec{P}-e\vec{A}(\vec{r},t)\right]^2}+eU(\vec{r}) \\ &=c\tilde{H}\left(\vec{P},\vec{r}\right)+eU(\vec{r})\doteq c\tilde{E}\left(\vec{P}\right)+eU(\vec{r})=E, \end{aligned} \tag{12.7}$$

for the electromagnetic field dressing the quantum particle, called photon, we obtained the properties

$$\begin{aligned} \left[\vec{P}-e\vec{A}(\vec{r},t)\right]^2 &= \vec{P}^2-2e\vec{P}\vec{A}(\vec{r},t)+e^2\vec{A}^2(\vec{r},t)=\text{const} \\ \vec{P}\vec{A}(\vec{r},t)=0,& \qquad \left|\vec{A}(\vec{r},t)\right|=\text{const.} \end{aligned} \tag{12.8}$$

With these conservative functions, the two wavefunctions (12.3) take the form

$$
\begin{aligned}
\psi(\vec{r},t) &= \frac{1}{(2\pi\hbar)^{3/2}}\int \varphi(\vec{P},t)\, e^{\frac{i}{\hbar}\left[\vec{P}\cdot\vec{r}-L(\vec{r},\dot{\vec{r}},t)t\right]}\mathrm{d}^3\vec{P} = \frac{1}{(2\pi\hbar)^{3/2}}\int \varphi(\vec{P},t)\, e^{\frac{i}{\hbar}\left\{\vec{P}\cdot\vec{r}-\left[\vec{P}\dot{\vec{r}}-H(\vec{P},\vec{r})\right]t\right\}}\mathrm{d}^3\vec{P} \\
&= \frac{1}{(2\pi\hbar)^{3/2}}\int \varphi(\vec{P},t)\, e^{\frac{i}{\hbar}\left[\vec{P}\cdot\vec{r}-\left(\vec{P}\dot{\vec{r}}-E\right)t\right]}\mathrm{d}^3\vec{P} \\
\varphi(\vec{P},t) &= \frac{1}{(2\pi\hbar)^{3/2}}\int \psi(\vec{r},t)\, e^{-\frac{i}{\hbar}\left[\vec{P}\cdot\vec{r}-L(\vec{r},\dot{\vec{r}},t)t\right]}\mathrm{d}^3\vec{r} = \frac{1}{(2\pi\hbar)^{3/2}}\int \psi(\vec{r},t)\, e^{-\frac{i}{\hbar}\left\{\vec{P}\cdot\vec{r}-\left[\vec{P}\dot{\vec{r}}-H(\vec{P},\vec{r})\right]t\right\}}\mathrm{d}^3\vec{r} \\
&= \frac{1}{(2\pi\hbar)^{3/2}}\int \psi(\vec{r},t)\, e^{-\frac{i}{\hbar}\left[\vec{P}\cdot\vec{r}-\left(\vec{P}\dot{\vec{r}}-E\right)t\right]}\mathrm{d}^3\vec{r},
\end{aligned}
\qquad \textbf{(12.9)}
$$

with the property

$$
\left(\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right)_{\mathrm{wave}} = \frac{\partial}{\partial\vec{P}}\left(\vec{P}\,\dot{\vec{r}}-E\right) = \dot{\vec{r}}\,, \qquad \textbf{(12.10)}
$$

which means a description of the particle matter distribution.

In the two-dimensional flavor space, we distinguish the up and down states of a quark,

$$
|\psi_u\rangle = \begin{pmatrix} \psi_u(\vec{r},t) \\ 0 \end{pmatrix}, \qquad |\psi_d\rangle = \begin{pmatrix} 0 \\ \psi_d(\vec{r},t) \end{pmatrix}. \qquad \textbf{(12.11)}
$$

With electric charge

$$
e_u = \frac{2}{3}e \ \text{ and } \ e_d = -\frac{1}{3}e\,, \qquad \textbf{(12.12)}
$$

in the linear electromagnetic space, and the flavor charges

$$
\mathbf{q}_u = \begin{pmatrix} q_u \\ 0 \end{pmatrix}, \qquad \mathbf{q}_d = \begin{pmatrix} 0 \\ q_d \end{pmatrix}, \qquad \textbf{(12.13)}
$$

in the flavor space. In this two-dimensional space, we define a basis of $2^2 - 1 = 3$ Hermitian operators, represented by the Pauli pseudospin matrices, CHAPTER 13

Conclusion

We reformulated quantum mechanics for particles as distributions of matter with intrinsic mass. First of all, we put on the same footing, the two empirical theories of Planck-Einstein and de Broglie, which describe oscillations in time and space of a quantum particle. In this way, we obtained a description of a quantum particle by two conjugate wave packets in two conjugate spaces with the coordinates and momentum, which was in perfect agreement with the Schrödinger equation only for the steady states. Essentially, to be in agreement with the fundamental Hamilton equations, in these wave packets, we had to consider the frequency proportional not to the Hamiltonian as in conventional quantum mechanics but to the Lagrangian. For an electron, this proportionality of the wavefunction frequency to the Lagrangian is easily understandable from Einstein's law of the photoelectric effect: the energy $E = \hbar\omega$ of a photon extracting an electron from an atom has to cancel the negative potential energy of this electron in the atom, $U\left(\vec{r}\right) < 0$, by providing the positive energy $-U\left(\vec{r}\right)$ to extract the electron, plus the kinetic energy T of this electron, which means a resonant process, with the frequency $\omega = \left(T - U\right)/\hbar$ of the electron time-oscillation. Using the relativistic Lagrangian, we obtained wave/group velocities equal to the wavefunction coordinate velocity, which means that these waves describe the motion of the wave packet as a whole, which further means that these waves represent the matter dynamics. More than that, this means that the integral of the matter density of this wave packet, as an amplitude, must be the same as the mass in the Lagrangian of the phase describing the dynamics of this wave, which is the mass quantization. The two conjugate wavefunctions are described by Schrödinger-like equations in the two conjugate spaces of the coordinates and of the momentum but with additional terms depending on velocity.

As a dark quantum particle is described by two conjugate wave packets in two conjugate spaces of the coordinates and momentum, with time-dependent phases proportional to the relativistic Lagrangian, we describe a charged particle interacting with an electromagnetic field by additional Lagrangian terms, with an electric potential conjugated to time, and a vector potential conjugated to the coordinates. From the wavefunction in the momentum space, we reobtained Lorentz's force as a function of electric and magnetic fields satisfying the Maxwell

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equations. For the two dynamic equations in the two coordinate and momentum spaces, we reobtained the two Dirac equations for a free particle of the quantum field theory, but with an additional relativistic function depending on the velocity and the physically understandable solution of a particle-antiparticle system in an electromagnetic field, as two finite distributions of matter and antimatter departing one another. Based on these equations, we reformulated the Fermi golden rule, including the velocity dependence of the matrix elements and of the density of states, and calculated the transition densities for the two possible processes, with spin conservation and with spin inversion.

We considered Dirac's formulation of general relativity of the physical space as a hypersurface in the total space, including a larger number of dimensions. On this basis, we describe the matter dynamics at the formation of a black hole. We obtained that, according to the general theory of relativity, the matter has not the tendency to concentrate at the black hole central part, as it is asserted in the conventional cosmology, but at the Schwarzschild boundary, which is physically more understandable: when a sufficiently large quantity of matter gravitationally concentrates inside the Schwarzschild boundary, the smaller central part of matter moves towards the larger matter part of the spherical crown surrounding the central region – in a gravitational interaction the lighter bodies move towards the heavier bodies. According to the general theory of relativity, the formation of a black hole begins with a bang, an explosion of the central part, with a velocity much larger than the light velocity, as the inner matter travels towards the Schwarzschild boundary but reaches this boundary only in an infinite time. We showed that at the formation of a black hole, a central quantum particle is crushed up in the momentum space, exploding in the coordinate space. Near the Schwarzschild boundary, a quantum particle is crushed up in the coordinate space, with a quasi-infinite momentum variation, as a force bringing the particle towards this boundary from the inside and from the outside of the black hole. A far outside the particle is attracted towards the black hole, but only up to three times the Schwarzschild radius. From the three times the Schwarzschild radius up to the Schwarzschild radius, the particle is repelled, reaching this radius with the null velocity and the null acceleration in an infinite time. In perfect spherical symmetry, no particle enters the black hole, and no particle exits the black hole. Of course, in the realistic case of a strongly perturbed black hole, as in collisions with outside celestial bodies, the Schwarzschild boundary is passed from the outside and from the outside. Accordingly, an old black hole, as the distribution of matter at the Schwarzschild boundary becomes thinner and thinner, finally dies, with this matter being spread in the surroundings by its collision with the surrounding bodies.

The black hole dynamics suggested that our universe is, in fact, a large black hole in the physical universe, spreading beyond its Schwarzschild limit as an infinite space inaccessible to our observations. This way, we understand the Big Bang and the inflation of our universe according to general relativity. Due to its large radius, we considered, according to the previous investigations, the acceleration of a body in the total gravitational field of our universe is ten orders of magnitude smaller than the gravitational acceleration on Earth, which means very small accelerations between any two observable distant bodies, thus behaving as quasi-inertial bodies. From the invariance of the time-space interval, we obtained the frequency variation of the light emitted by atoms placed at different distances in our universe, as the gravitational effect in the visible universe of matter remained after a long Big Bang-Inflation process. For a quantum particle in our universe, we calculated the momentum as a function of radius by two methods, taking this particle in the proper time, and in the local time, and the total gravitational force in the universe. For a central particle, at the beginning of our universe, this force takes an infinite value, corresponding to the Big Bang. For a quantum particle fluctuating near the Schwarzschild boundary, this force takes large values, bringing the particle back to this boundary. Of course, after Big Bang, the total gravitational field of our universe has been strongly perturbed by nuclear and electric forces, as it is described by the present cosmology and astrophysics. Unlike the conventional science regarding our universe, Big Bang does not refer to the total universe with the total matter initially concentrated at a point but to the central part of a large agglomeration of matter, which, taking a spherical symmetry inside the Schwarzschild boundary, provokes an explosion of the central part, as the inner part of this matter receives gravitational energy towards the Schwarzschild boundary. However, in strong collisions, with external forces much stronger than the gravitational ones, this gravitational energy is dissipated, a system of bodies that we call the internal universe remaining inside the Schwarzschild boundary. The cosmological redshift can be understood according to general relativity only by taking into account the gravitational attraction between the bodies of the internal universe as a frequency shift due to the larger velocity of the observer, supposed to be placed closer to the universe centre, than the slower velocity of a light emitting body placed at a far distance from this centre. When, by gravitational attraction, the internal universe will take a sufficiently spherical symmetry, a new Big Bang-Inflation process will occur, sending a part of the inside matter towards the Schwarzschild boundary with a velocity much larger than the light velocity and a new internal universe will be formed by the matter bodies remaining inside. In this time, the Schwarzschild boundary will become thinner and thinner by outside collisions, being finally dispersed in the infinite everlasting universe – the definite end of our universe.

REFERENCES

[1]. K.H. Li, "Physics of open systems", *Phys. Rep.,* vol. 134, no. 1, pp. 1-85, 1986.
http://dx.doi.org/10.1016/0370-1573(86)90101-8

[2]. G. Lindblad, "On the generators of quantum dynamical semigroups", *Commun. Math. Phys.,* vol. 48, no. 2, pp. 119-130, 1976.
http://dx.doi.org/10.1007/BF01608499

[3]. A. Săndulescu, and H. Scutaru, "Open quantum systems and the damping of collective modes in deep inelastic collisions", *Ann. Phys.,* vol. 173, no. 2, pp. 277-317, 1987.
http://dx.doi.org/10.1016/0003-4916(87)90162-X

[4]. A. Sandulescu, and E. Stefanescu, "New optical equations for the interaction of a two-level atom with a single mode of the electromagnetic field", *Physica A,* vol. 161, no. 3, pp. 525-538, 1989.
http://dx.doi.org/10.1016/0378-4371(89)90440-8

[5]. E. Stefanescu, A. Săndulescu, and W. Greiner, "Quantum Tunneling in Open Systems", *Int. J. Mod. Phys. E,* vol. 2, no. 1, pp. 233-258, 1993.
http://dx.doi.org/10.1142/S0218301393000078

[6]. E. Stefanescu, A. Sandulescu, and W. Greiner, "Analytical model for quantum tunnelling with dissipation through a fission-like barrier", *J. Phys. G Nucl. Part. Phys.,* vol. 20, no. 5, pp. 811-828, 1994.
http://dx.doi.org/10.1088/0954-3899/20/5/014

[7]. A. Isar, A. Sandulescu, H. Scutaru, E. Stefanescu, and W. Scheid, "Open Quantum Systems", *Int. J. Mod. Phys. E,* vol. 3, no. 2, pp. 635-714, 1994.
http://dx.doi.org/10.1142/S0218301394000164

[8]. E. Stefanescu, E. Halmagean, and A. Sandulescu, "Dissipative Effects in Quantum Tunneling, NATO-ASI Series, Series B", *Physics,* vol. 342, p. 517, 1995.

[9]. E. Stefanescu, W. Scheid, A. Sandulescu, and W. Greiner, "Cold fission as cluster decay with dissipation", *Phys. Rev. C Nucl. Phys.,* vol. 53, no. 6, pp. 3014-3021, 1996.
http://dx.doi.org/10.1103/PhysRevC.53.3014 PMID: 9971289

[10]. E. Stefanescu, R.J. Liotta, and A. Sandulescu, "Giant resonances as collective states with dissipative coupling", *Phys. Rev. C Nucl. Phys.,* vol. 57, no. 2, pp. 798-805, 1998.
http://dx.doi.org/10.1103/PhysRevC.57.798

[11]. E. Stefanescu, and W. Scheid, "Superradiant dissipative tunneling in a double p–i–n semiconductor heterostructure with thermal injection of electrons", *Physica A,* vol. 374, no. 1, pp. 203-210, 2007.
http://dx.doi.org/10.1016/j.physa.2006.07.014

[12]. *Quantum Dynamical Semigroups.*. Springer-Verlag: Berlin, Heidelberg, New York, London, Paris, Tokyo, 1987.R. Alicki, and K. Lendi, Lecture Notes in Physics.

[13]. E. Stefanescu, A. Sandulescu, and W. Scheid, "The Collisional Decay of a Fermi System Interacting with a Many-mode Electromagnetic Field", *Int. J. Mod. Phys. E,* vol. 9, no. 1, pp. 17-50, 2000.

http://dx.doi.org/10.1142/S0218301300000039

[14]. E. Stefanescu, and A. Sandulescu, "Microscopic Coefficients for the Quantum Master Equation of a Fermi System", *Int. J. Mod. Phys. E,* vol. 11, no. 2, pp. 119-130, 2002.

http://dx.doi.org/10.1142/S0218301302000739

[15]. E. Stefanescu, and A. Sandulescu, "Dynamics of a Fermi System in a Blackbody Radiation Field", *Int. J. Mod. Phys. E,* vol. 11, no. 5, pp. 379-386, 2002.

http://dx.doi.org/10.1142/S0218301302000910

[16]. E. Stefanescu, W. Scheid, and A. Sandulescu, "Non-Markovian master equation for a system of Fermions interacting with an electromagnetic field", *Ann. Phys.,* vol. 323, no. 5, pp. 1168-1190, 2008.

http://dx.doi.org/10.1016/j.aop.2008.01.009

[17]. E. Stefanescu, "Open Quantum Systems of Particles and Principle 2 of Thermodynamics", *J. Phys. Conf. Ser.,* vol. 413, 2013.012038.

http://dx.doi.org/10.1088/1742-6596/413/1/012038

[18]. *"Longitudinal Quantum Heat Converter, Inventors: Eliade Stefanescu and Lucien Eugene Cornescu,"*, US 20090007950 (US Patent Office, Jan. 08, 2009).

[19]. *"TRANSVERSAL QUANTUM HEAT CONVERTER, Inventors: Eliade Stefanescu and Lucien Eugene Cornescu,"*, US 20100019618 (US Patent Office, Jan. 28, 2009).

[20]. *"QUANTUM INJECTION SYSTEM, Inventors: Eliade Stefanescu and Lucien Eugene Cornescu,"*, US 20090007951 (US Patent Office, Jan. 08, 2009).

[21]. E. Stefanescu, "Master equation and conversion of environmental heat into coherent electromagnetic energy", *Prog. Quantum Electron.,* vol. 34, no. 6, pp. 349-408, 2010.

http://dx.doi.org/10.1016/j.pquantelec.2010.06.003

[22]. E. Stefanescu, *Open Quantum Physics and Environmental Heat Conversion into Usable Energy.,* vol. Vol. I. Bentham Science Publishers: Singapore, 2014.

[23]. E. Stefanescu, *Open Quantum Physics and Environmental Heat Conversion into Usable Energy.,* vol. Vol. II. Bentham Science Publishers, Singapore, 2017.

http://dx.doi.org/10.2174/97816810841901170201

[24]. P.A.M. Dirac, *General Theory of Relativity.* John Wiley & Sons: New York, London, Sydney, Toronto, 1975.

[25]. P.A.M. Dirac, *The Principles of Quantum Mechanics.* Oxford at the Clarendon Press, 1958.

http://dx.doi.org/10.1063/1.3062610

[26]. A. Einstein, *The Meaning of Relativity.* Chapman and Hall: London, New York, Tokyo, Madras, 1991.

[27]. E. Stefanescu, "Unitary Theory of Quantum Mechanics and General Relativity", *European Journal of Applied Sciences,* vol. 9, no. 4, pp. 108-153, 2021.

[28]. E. Stefanescu, "Dynamics of a quantum particle interacting with an electromagnetic field", *European Journal of Applied Sciences,* vol. 9, no. 6, pp. 463-501, 2021.

[29]. E. Stefanescu, "Mass and scattering of a quantum particle as a relativistic distribution of matter", *European Journal of Applied Sciences*, vol. 10, no. 2, pp. 430-447, 2022.

[30]. E. Stefanescu, "Quantum Mechanics as a theory based on the general theory of relativity", *Annals of the Academy of Romanian Scientists - Series on Physics and Chemistry Sciences,*, vol. 7, no. 1, pp. 7-52, 2022.

[31]. E. Stefanescu, "Fermi's Golden Rule and the Scattering of a Quantum Particle as a Distribution of Matter", *European Journal of Applied Sciences*, vol. 10, no. 4, pp. 660-674, 2022.

[32]. E. Stefanescu, *Open Quantum Physics and Environmental Heat Conversion into Usable Energy.*, vol. Vol. 3. Bentham Science Publishers: Singapore, 2022.

[33]. A. Das, *Lectures on Quantum Field Theory*. World Scientific, New Jersey, London, Singapore, Beijing, Shanghai, Hong Kong, Taipei, and Chennai, 2008.
http://dx.doi.org/10.1142/6938

[34]. M. Thomson, *Modern Particle Physics.* Cambridge University Press, 2013.
http://dx.doi.org/10.1017/CBO9781139525367

[35]. L. Smolin, *Einstein's Unfinished Revolution – The Search for What Lies Beyond the Quantum.* Penguin Books, 2020.

[36]. R.P. Feynman, *Feynman's Lectures on Gravitation.* Edited by Brian Hatfield, (CRC Press, Taylor & Francis Group, Boca Raton, London, New York, 2018.

[37]. *Observable universe*. https://en.wikipedia.org/wiki/Observable_universe

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