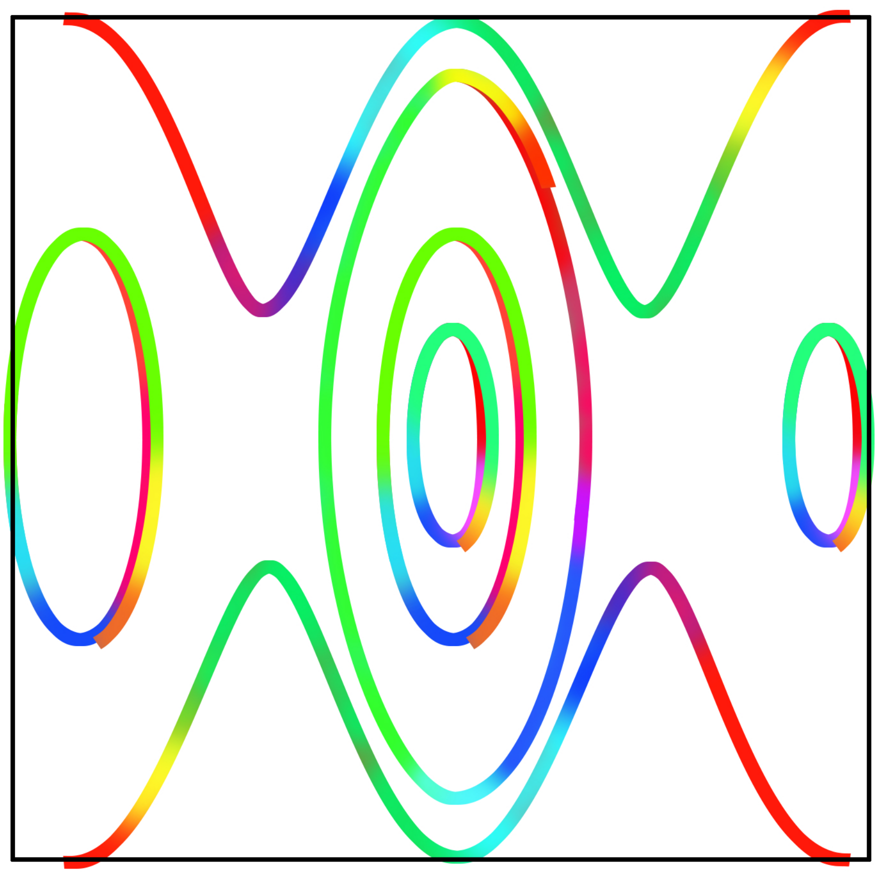
ORDINARY DIFFERENTIAL EQUATIONS AND APPLICATIONS I:

WITH MAPLE EXAMPLES



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Ordinary Differential Equations and Applications I: With Maple Examples

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Ordinary Differential Equations and Applications I: with Maple Examples

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FOREWORD

I strongly endorse this exceptional book on the subject of differential equations. It covers all aspects of the field. It has a solid theoretical foundation and an applied focus, with many practical examples. It demonstrates how to program them using Maple, which is a leading mathematical software; and finally, it demonstrates how to generate graphics that clearly represent the nature of solutions and provide deep insights into them. All of these aspects are essential in the use of differential equations in modern mathematics, science, and technology. Thus, the book is equally useful for mathematicians, scientists, and engineers. As engineers should have some understanding of the theory of differential equations, also mathematicians should be able to program and generate graphical results.

This volume is especially valuable because it presents all of these aspects in an integrated fashion. It is written by a true expert in the field, an experienced teacher who has also carried out significant applied research. As a master teacher, Dr. Oyelami presents the material in a simple, straightforward, easy-to-follow manner. As an expert researcher, he knows first-hand the power of differential equations as a modeling tool, and his love for the field is clearly visible. The volume is also comprehensive in its coverage, especially in the areas of differential equations of the greatest practical interest. The students who study this material will be thoroughly prepared for employment in technical fields that use differential equations for modeling purposes. Such a student will also find the book to be a valuable continuing reference, both for its clear theoretical presentations and its useful and generalizable computer codes.

Christopher Thron
Associate Professor of Mathematics,
Texas A&M University, Central,
Texas, USA

ENDORSEMENT

I have thoroughly gone through this book, which can be considered to be a compendium of knowledge on Differential Equations at the Undergraduate levels in all ramifications. The book presents poignantly insightful views on quantitative and qualitative modeling, as it unleashes the tremendous power of differential equations techniques, with applications on current multifarious trends, including population dynamics, spread of viruses and diseases and neural networks.

This book places the generally neglected implementation aspects of mathematical results on the front burner, with special implementations on the platform of Maple. In the above regard, the contributions of this book are exceptional and unprecedented. In terms of scope and diversity, the reader will be surprised by the unfathomable depth of knowledge and broad horizon of the author on the mathematical modelling of continuous processes by the deft deployment of differential equations.

The book must be highly acclaimed for its balanced coverage of the theory, applications, and computational issues of differential equations and their solutions. It gives an effective exposition of differential equations and concepts with functional analytic support, as needed, with meticulously chosen examples, exercises and extensive use of Maple, currently regarded as the best mathematical software. This is the main thrust of the book, as it encompasses and emphasizes current trends of modern computational tools in enhancing the effectiveness of differential equations as an indispensable and core tool for modelling of processes that exist in the continuum.

On the other hand, the book reinforces the reader’s understanding of ordinary differential equations, which, on the other hand, simulates and enhances the readers’ interest and curiosity about the immense modelling possibilities on ordinary differential equations platforms.

This book vividly brings to the fore, the inconvertible fact that, for the most part, ordinary differential equations cannot be precluded in the modelling of real-life phenomena. This being an exceptionally well-crafted book with an abundance of realistic, well-researched examples, illustrations and exercises, will enliven discussions of ordinary differential equations, techniques and key modelling objectives that the reader will likely encounter in undergraduate courses and much more.

In view of the aforementioned attribute coupled with its lucid presentation, novelty of the abstract of each chapter and emphasis on digital implementations, this book deserves the highest recommendation. The book is a ‘must-read ‘and ‘unputdownable’.

Professor Ukwu Chukwunenye
Functional Differential Equations,
Control theory & Industrial Engineering Specialist,
Department of Mathematics,
University of Jos, Jos, Nigeria

Comments from Renowned Scientists

‘‘I have gone through the whole book. It is simple, clear and easy to read and understand .I have no doubt in my mind that the book is a must for all students of Mathematics in Tertiary Institutions’’

Professor M O. Ibrahim
University of Ilorin,
Ilorin and former President of Mathematics Association of Nigeria (MAN)

‘‘The book will be a very good choice for both professionals across all fields of endeavours. The fact that the book does not assume familiarity with some basic mathematical concepts is an incentive in its appeal to those who have been out of school for some time. These qualities will increase its sellable quality in the market place as well as a recommendation to students on mathematical courses’’.

Professor Emeritus A.A. Asere
Department of Mechanical Engineering,
Obafemi Awolowo University, Osun,
Nigeria

PREFACE

Ordinary differential equations as a branch of Mathematics has grown from a tiny mustard seed to a giant tree over the centuries. Its outgrowth could be said to have originated from simple problems of finding solutions to equations involving rates of change of the dependent variable with respect to the independent variable x or time t. For example, finding the rate at which a balloon can be inflated or deflated.

Today, human interests and logistics are diverse and go beyond this datum level. It is above a mere study of fluxion theory of Isaac Newton that could be said to be the foundation stone of differential equations.

This textbook is an encyclopedia of techniques for finding solutions to differential equations. It was developed when lecturing students and researching at the Abubakar Tafawa Balewa University, Bauchi; Kaduna State University, Kaduna, Nigerian; Nile University, Baze university, University of Abuja all in Abuja and the Plateau State University Bokkos , all in Nigeria.

This book comprises seven chapters and it is on ‘scalar differential equations’. The chapters are designed so that beginners in the field of study who have little or no background on the course can easily understand the book. This requirement is met by deployment of lucid and self-instructional language and utilization of scintillating examples throughout the book as well as illustration using Maple modeling and simulation software. Maple and MapleSim software are reconnoitered for finding symbolic solutions to problems in ODEs and simulation of engineering systems. Maple examples on how to find the analytic solutions to the ODEs problems, and plotting and animation of solution paths in 2D and 3D forms are presented.

Furthermore, among pages of ‘freshman’ Chapters are the treatment of variable separable, exact equations, the method of undetermined coefficients and variation of constant parameters method. The celebrated Green’s function technique as applicable to the boundary value problems (BVP) has also been presented with several examples from physical, biological and engineering problems. Chapter six introduces an algebraic structure, the vector space, concept of linear transformation and the differential operator ‘D method’; this method is instrumental to finding particular integrals to the differential equations. Chapter seven is on solutions of differential equations by power series.

Every part of the chapters in this textbook contains preambles without assuming students' familiarity with some basic mathematical concepts. Hence it will prove to be a valuable and supplementary textbook for other courses in Mathematics and Engineering.

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My profound gratitude goes to Professor P. Smoczynski of the Department of Mathematics and Statistics, Simon Fraser University, Canada who first introduced me to Differential Equations and sustained my interest in the field. I am indebted to Professor Styr University of Botswana and late Professor Olaofe, University of Ibadan both of them taught me Numerical Analysis at undergraduate and postgraduate levels respectively. In ship of thanks are Dr. Ukwu Chukwunenye my Lecturer in the University of Jos Nigeria; Mr. Salam Mukaila a friend, late Professor M. Ibiejugba Koji State University Ayingba, Professor G. Abimbola and Professor M. O Ibrahim University of Ilorin, Nigeria; Professor Christopher Thron and Gwenda Lynn Anders, Texas A&M University-Central Texas, USA; late Professor P.C. Ram, and Professor M S.Sesay , Abubakar Tafawa Balewa University Bauchi, Nigeria.

Furthermore, I am grateful to: Professor S O Ale, National Mathematical Centre, Abuja, Nigeria; Professor A.A. Asere, Obafemi Awolowo University Ile-Ife Nigeria; Late Professor D.D. Bainov, Medical University Sofia Bulgaria; Professor Olusola Akinyele, Bowie State University, Maryland USA for their exposure to Impulsive differential equations and my mentor, Professor Emeritus Trench William Trinity College USA , and Professor R.A.T. Solarin and Professor Stephen Onah, the former Directors and Chief Executive of National Mathematical Centre(NMC), Abuja, Nigeria respectively. Professor Promise Mebine, the Present Director and Chief Executive of NMC. I am grateful to Professor Femi Taiwo Obafemi Awolowo University Ile-Ife. We are Co-Trainer for Maple Software across some Nigerian Universities. I am greatly indebted to the former Vice Chancellor, Plateau State University Bokkos, Professor Danjuma Sheni who through TETfund Research grant for preparation of this book. I am also grateful to Colleagues at the National Mathematical Centre, University of Abuja, Abuja and the Baze University all in Abuja, Nigeria.

I am grateful to the above academicians for their technical advice, thought provoking suggestions and eagle-eye proofreading of the whole manuscript. I am grateful to Mr. Anthony Oluloye of Tangier Company, who provided me with Maple 17, 18, 2015,2018,2019,2020,2021,2022,2023, MapleSim 7 and MapleSi 2023 gratis.

Finally, I must not forget my family especially my wife Keith E. Oyelami and my children; Moses, Ruth, Victoria, Miracle, David and Hannah for their contribution in making the publication of this book successful. I am grateful God bless you all.

DEDICATION

Dedicated to God Almighty, most merciful, the author of life, the giver of knowledge, wisdom and understanding. The procreator, sustainer, and annihilator of all life processes. God is the greatest problem solver who can solve a problem in an infinitely many ways.

CHAPTER 1

Introduction to Ordinary Differential Equations

Abstract: This section introduces the sensing and perception of the soft robots. First, the sensor of the soft robots are classified. Then, the state of the soft robot's sensing and perception model is introduced, *i.e.*, passive mode, semi-active mode, active mode and interactive perceptive mode. Moreover, according to the sensing and perception mode, several sensing and perception technologies are presented. In the end, several challenges regarding the sensing and perception of the soft robots are summarized.

Keywords: Current flow in LRC circuits, Directional field, Exact, Functional, Isoclines for parabola, Lotka- Volterra model, Newton's rate of cooling equation, Orthogonal trajectories, Quasilinear, Pendulum oscillators, Riccati differential equations, Rectangular hyperbola, Van de Pol.

INTRODUCTION

The branch of mathematics that studies equations involving derivatives of unknown functions is called differential equations. There are two classes of such equations that are classified according to the number of unknown variables involved.

A differential equation is a relationship between an independent variable, x x and dependent variable y, and one or more derivatives of y with respect to x.

Differential equations with a single unknown variable are called ordinary differential equations (ODEs). ODEs find applications in mathematical physics, electrical engineering, and mechanical engineering, for example in the vibration of strings [2, 4-6].

The general form of linear ordinary differential equations can be written as:

$$a_o(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_n(x)y = f(x) \tag{1.1}$$

Where $a_0, a_1, \ldots, a_n$ and $f(x)$ are continuous functions of x .The basic theory of differential equations gives a thorough characterization of solution of (1.1). A second type of linear equation consists of partial differential equations (PDEs)

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which are commonly found in theoretical physics. They are concerned with the study of equations involving partial derivatives of several independent variables [3, 7-10].

Here are some examples of commonly found (PDEs):

Example 1.1

The Poisson equation

$$\Delta u = 0 \ or \ \Delta u = -4\pi q \tag{1.2}$$

where $\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^k}$ is the Laplace operator.

This appears in the theory of gravitation and electromagnetism as the potential equation. The term q represents the field sources (mass in gravitation and charge in electromagnetism). In hydrodynamics of incompressible and irrational fluid, equation (1.2) is frequently encountered with.

Example 1.2

The one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \ a = \frac{T}{\rho} \tag{1.3}$$

where T = tension of vibrating string. ρ = density of the string and u = length of the string.

Example 1.3

The Schrödinger's' equation is found in the theory of thermodynamics and statistical mechanics and it is represented by the equation

$$i\hbar \frac{\partial \psi}{\partial x} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \tag{1.4}$$

where $\hbar$ =Planck's constant, $\hbar = \frac{h}{2\pi}$, m =mass of the particle, ψ is the wave function and V the potential. Finally, as an instance of Example 1.1.

Consider the Laplace's equation in 3-diamension:

$$\Delta^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = 0 \tag{1.5}$$

The equations in (1.4) & (1.5) are popularly found in electrodynamics and electromagnetism. The partial differential equations (PDEs) in the equations (1.2) and (1.4) will reduce to ordinary differential equations if the problem is in one dimensional form. There are other classes of differential equations with underlying equations being subject to shocks or rapid changes; such equations are said to be impulsive differential equations (IDEs) [8-10]. In this book, we will not consider PDEs and IDEs.

There are several methods for reducing PDEs to ODEs problems, but for this book, our attention will mainly be focused on ODEs.

Formulation of Differential Equations

In theoretical physics and dynamical systems one encounters many differential equations, some of which are easily solvable using available known techniques [1, 3, 7-9].

In this book, we shall consider different types of ordinary differential equations and build up various kinds of techniques for solving them. For this purpose, we have to introduce some basic definitions.

Definition 1.1

An equation of the form

$$F\left(x, y, y', \ldots, y^{(n)}\right) = 0 \tag{1.6}$$

where $y = y(x)$ is the unknown or sought for function is, said to be a differential equation of order n .In other words, the order of a differential equation can be defined as the largest positive integer n for which the n derivatives of the unknown function appears in the equation.

CHAPTER 2

Solutions of First Order Differential Equations and Applications

Abstract: In this chapter, first order linear differential equations together with the nonlinear Bernoulli equations are considered and methods for obtaining their solutions discussed. The methods are applied to find solutions to models in the vibration of LR electrical circuits, radioactive dating, population dynamics, chemical reactions, epidemiological and pollution problems.

Keywords: Chemical reactions, Epidemiological, Population dynamics, Pollution problems, Radioactive dating,Vibration of LR electrical circuits

INTRODUCTION

This chapter will consider the first order linear differential equations and associated practical problems. Linear differential equations are more frequently encountered equations in ordinary differential equations [1-3]. There are many types of methods used in solving them. We shall consider some selected ones and their solution methods. The first order linear differential equations to be considered are: vibration of LR electrical circuit, radioactive dating, population dynamics, chemical reactions, epidemiological problems, and pollution problems [4-7].

First-order Linear differential Equations

Consider the differential equation:

$$(\mathrm{LO}^1)\quad a_0(x)\frac{dy}{dx} + a_1(x)y = h(x), a_0(x) \neq 0$$

where $a_0(x), a_1(x)$ and $h(x)$ are continuous functions of x. This is an example of a class of non-homogeneous differential equations. It can be rewritten in the form:

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.... $$\frac{dy}{dx} + \rho(x)y = \phi(x) \qquad (2.1a)$$

This equation (2.1a) is also referred to as a Leibnitz's differential equation.

In the homogeneous case, $\phi(x) = 0$

Therefore:

.... $$\frac{dy}{dx} + p(x)y = 0 \qquad (2.1b)$$

Thus $\frac{dy}{dx} + p(x) = Q(x);$ integrating we get $\ln y + \int^x p(s)ds = C,$ where C is constant .Therefore, $y(x) = C\exp(-\int^x p(s)ds)$.

Now let's solve for the non-homogeneous case. These yields:

$$\frac{dy}{dx} + p(x)y = Q(x)$$

This equation can be transformed into an exact differential equation by multiplying it by the integrating factor (I.F) $e^{-\int \rho(x)dx}$.

Thus

.... $$(\frac{dy}{dx} + p(x)y)e^{-\int p(x)dx}y = Q(x)e^{-\int p(x)dx} \qquad (2.2)$$

which yields:

$$d(e^{-\int p(x)dx}y) = Q(x)e^{-\int p(x)dx} \qquad (2.3)$$

Integration yields:

$$y(x) = (\int Q(s)e^{-\int p(x)dx}ds)e^{-\int p(x)dx} + C, C \text{ is constant} \qquad (2.4)$$

That is:

$$y \times I.F = \int (Q(x) \times I.F) dx + C$$

This formula is Leibnitz's formula for a differential equation.

Example 2.1

Find the general solution of:

$$\frac{dy}{dx} = \frac{1}{e^y - x}$$

Solution

$$\frac{dx}{dy} = e^y - x$$

The equation is linear or Leibnitz type and the integrating factor I.F:

$$I.F = e^{\int dy} = e^y$$

Therefore

$$\frac{dx}{dy} e^y + x e^y = e^y e^y = e^{2y}$$

$$\int d(xe^y) = \int e^{2y} dy$$

$$xe^y = \frac{1}{2} e^{2y} + C, \ C = constant$$

Therefore

$$x = \frac{1}{2} e^y + Ce^{-y}$$

is the general solution to the given equation.

CHAPTER 3

Second Order Differential Equations and Applications to some Models

Abstract: Second order differential equations and methods for solving them are studied. Methods considered are: Undetermined coefficients, Green's and Wronskian, principle of superposition of solutions and variation of constant parameters. Also elucidated upon are: Construction of Green's functions and applications to boundary value problems, Cauchy-Euler, Lagrange and Clairaut equations. Many solved examples and presents which include Maple ones.

Keyword: Cauchy-Euler equations, Clairaut equations, Green's function, Lagrange, Maple solved examples, Undetermined coefficients, Variation of constant parameters, Wronskian,

SECOND ORDER DIFFERENTIAL EQUATIONS

Many problems in real life can be modelled using ordinary differential equations (ODEs). In Chapters one and two, a variety of methods for solving first order Linear equations are presented. This chapter we are extending our study to second order linear differential equations. Methods for solving them such as undetermined coefficient, variation of constant parameters and green's function will be presented with applications to some models.

The general differential equation of second order may be written as:

$$a_0 x^{//} + a_1 x^{/} + a_2 x \;=\; f(x) \tag{3.1}$$

where a_0, a_1 and a_2 are continuous functions of x .

The differential equation in equation (3.1) is said to be homogenous, if on the other hand, the right-hand side is identically zero, *i.e.* $f(x) = 0$.

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Definition 3.1

The solution y_c of the homogeneous equation in equation (3.1) is called complementary function. The solution y_p of the non-homogeneous equation in equation (3.1) is termed the particular solution.

Undetermined Coefficients Method

This section considers a popular technique called undetermined coefficients method. We shall apply the method to obtain solutions of second order differential equations. As we progress in study, the method will be generalized to higher order differential equations. This method has its own weakness just like other methods. This will be pointed out as we progress in our study.

We consider the equation (3.1) where a_0, a_1 and a_2 are assumed to be constants and $f(x) = 0$. In other words, let (3.1) be a constant coefficient homogeneous equation.

Suppose that $y_c = e^{\lambda x}, \lambda = constant$ is the complementary function ([3-6]). Then y_c must satisfy equation (3.1), that is:

$$a_o\lambda^2 e^{\lambda x} + a_1\lambda e^{\lambda x} + a_2 e^{\lambda x} = 0 \tag{3.2}$$

Divide the equation (3.2) by $e^{\lambda x}$ yields:

$$a_o\lambda^2 + a_1\lambda + a_2 = 0$$

Now let:

$$p(\lambda) = a_0\lambda^2 + a_1\lambda + a_2 = 0 \tag{3.3}$$

Note that the contribution of $e^{\lambda x}$ cancel out, or equivalently, $p(\lambda) = a_0\lambda^2 + b_1\lambda + b_0 = 0$ where $b_1 = \frac{a_1}{a_o}$, $b_2 = \frac{a_2}{a_o}$.

The polynomial $p(\lambda)$ is called the characteristic polynomial associated with the homogenous past of equation (3.1).

There are three situations regarding the root of $p(\lambda)$ ([3-6]):

Case 1: When the roots are real and distinct $(b_1^2 > 4b_2)$

1. When the roots are complex conjugate $b_1^2 < 4b_2)$

2. When the roots are equal$(b_1 = 4b_2)$.

The general solutions of the equation (3.1) can be obtained for the three distinct cases as:

Case I: $y = C_1e^{\lambda_1 x} + C_2e^{\lambda_2 x}$, where C_1 and C_2 are arbitrary constants, which may be determined from initial conditions. Verify this claim!

Case II

$$y = C_1e^{i\lambda_1 x} + C_2e^{i\lambda_2 x}$$, where:

$\lambda_1 = a + ib,\ \lambda_2 = \lambda_1 = a - ib,\ a,b\ are$ real numbers i is the imaginary unit.

Using the familiar De Movre formula:

$$e^{iQ} = \cos Qx + i\sin Qx$$

yields:

$$\begin{aligned} y &= C_1e^{i\lambda_1 x} + C_2e^{i\lambda_2 x} = C_1e^{ax}(\mathrm{Cos}\,bx + iSinbx) + C_2e^{ac}(Cosbx + iSinbx) \\ &= (C_1 + C_2)e^{ax}\cos bx + (C_1 - C_2)e^{ax}\sin bx \\ &= e^{ax}[A\cos bx + B\sin bx] \end{aligned} \tag{3.5}$$

where A and B are complex constants.

CHAPTER 4

4.0 Fourier Series and Applications

Abstract: In this chapter, Fourier series is introduced for functions which are Riemann integrable and are of bounded exponential growth. Orthogonal relations; least square error, completeness relation and Riemann- Lebesgue theorem are also considered. The Fourier series is applied to obtain a series solution to some periodic boundary value problems. Also provided are Maple examples for applications of Fourier series to ordinary differential equations.

Keywords: Bounded exponential growth, Boundary value problems, Completeness relation, Differential equations, Fourier series, Least square error, Periodic solutions, Riemann- Lebesgue theorem

INTRODUCTION

The theory of Fourier series was propounded by Jean Baptize Fourier (1763 - 1830), in his book on the conduction of heat, which appeared in 1822 in Paris. The idea of Fourier series is the expansion of a function defined on an interval $-\pi \leq x \leq \pi$ into an infinite trigonometric series involving sine and cosine terms ([2-5]).

A rigorous study of Fourier series was carried out by Dirichlet and later by Riemann and Lebesgue who laid the foundation of the theory. Accordingly, Lebesgue gives a condition for the existence of the Fourier series based on the Lebesgue integrability or summability of $f(x)$, see Mesohata [4].

At the end of this section, solutions of periodic boundary value problems will be solved by the Fourier series technique.

Conditions for Fourier Expansion

The studies of Fourier expansion are usually restricted to a class of functions that are absolutely integrable and of bounded exponential growth. These terms are explained in the following definition:

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Definition 4.1

a. A function is said to be absolutely integrable in the interval

[a, b] if: $\int_a^b |f(x)|\,dx < +\infty$.

b.

$F(x)$ is of bounded exponential growth of if there exist two positive constants m and α such that

$|f(x)| \leq Me^{\alpha x}$

Any function that satisfies the definition 4.1 has a Fourier series expansion.

Formulation

Consider a function $f(x)$ defined in a closed interval $-\pi \leq x \leq \pi$. Then the Fourier series of $f(x)$ is defined as:

$$f(x) = \frac{A_0}{2} + \sum_0^\infty A_k coskx + \sum_0^\infty B_k sinkx \tag{4.1}$$

where

$$A_o = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(s)ds \tag{4.2}$$

$$A_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(s)\cos(ks)ds \tag{4.3}$$

$$B_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(s)\sin ks\, ds \tag{4.4}$$

The coefficients A_0, A_k and B_k in the equation (4.1) are called the Fourier coefficients. The question of existence of the equation (4.1) is stipulated in the Dirichlet Theorem:

1. That $f(x)$ must have finite extrema, that is, finite maximum or minimum.

2. It must be periodic and piecewise continuous. At the point of discontinuity *x*; $f(x) = \frac{1}{2}\left(f(x-0) + f(x+0)\right)$, the average value of the right and left limits of $f(x)$ at x is the point of discontinuity (Curtain and Pritchard [13,pp.189]).

Special kinds of Fourier series can be obtained from the equation (4.1) when A_k and B_k are zero. There are odd and even Fourier series corresponding to $f(x)$, respectively.

If $f(x)$ is an odd function, that is, $f\ (-x)\ =\ f(x)$ the Fourier series of $f(x)$ is odd, that is:

$$f(x) = \frac{A_o}{1} + \sum_{k=o}^{\infty} B_k \sin kx \tag{4.5}$$

The cosine term in this case disappears. If $f(x)$ is an even function, that is, $f\ (-x)\ =\ f(x)$ then:

$$f(x) = \frac{A_o}{2} + \sum_{k-o}^{\infty} A_k \cos kx \tag{4.6}$$

We note that the Sine term disappears in this case.

Orthogonal and Orthonormal Relations

$$\int \begin{Bmatrix} Cos\,kx\,Sin\,lx \\ Cos\,kx\,Cos\text{olx} \\ \text{Sin\~kx\~Sin\~lx} \end{Bmatrix} dx = \delta_{ik} \ = \begin{cases} 1 if\ l = k \\ 0 if\ l \neq k \end{cases} \tag{4.6b}$$

CHAPTER 5

Operational Calculus Approach for Solving Ordinary Differential Equations

Abstract: Laplace transform method is studied in this chapter. It is an operation calculus method for finding solutions to differential equations, especially solutions to models in engineering. Problems solved using the Laplace transform are: LRC electrical problems with constant voltages and n-th order linear differential equations. In real life, systems may be governed by combinations of continuous and discrete characteristics, referred to as hybrid systems. In order to handle such systems effectively, the Laplace transform of discrete systems is also studied to complement the continuous systems.

Keywords: Hybrid systems, Laplace transform, LRC electrical problems, Maple examples, Milne transform, Operation calculus.

INTRODUCTION

Operational calculus (OPC) could be said to have been introduced in London in 1899 by an English Physicist, Heaviside O (1850-1925) in his electromagnetic theory. He applied it to ordinary differential equations in connection with electrotechnical problems.

Operational calculus has become an indispensable tool in physics, mathematics, and technology. In particular, it is used for theoretic investigations concerning boundary value problems. OPC has infiltrated several areas in ordinary and partial differential equations.

Examples of OPC are: Laplace, Fourier, Melline, Hilbert transforms, *etc*. (See Musky [7], Brain [3], Dass and Rajnish [4], John [6], Oyelami and Ale (See also [8-9]). These textbooks are richly endowed with solved problems using (OPC) approach. Nevertheless, only Laplace and Milne transforms will be discussed in this chapter.

Laplace Transform

We define the Laplace transform of a function $f(t)$ as a function, $F(p)$ and it is given as:

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$$L\{ f(t)\}(p) = \int_0^{\infty} e^{pt} f(t)dt \quad (\mathrm{Re}\, p > 0) \tag{5.1}$$

$f(t)$ is a complex-valued function and p is the complex number. The assumption that $\Re e\ p > 0$, that is, the real part of p is positive is to ensure that the integral in (5.1) converges. Another condition for the convergence of the integral is that $f(t)$ must have a bounded exponential rate of growth (cf, Musky [7], p163) this simply means that:

$$\| f(t) \| \leq m e^{s_0 t} \tag{5.2}$$

where m and s_0 are constants.

Polynomials are excellent examples of functions with bounded exponential rate of growth.

Let us apply the transform to some specific examples:

Example 5.1

$$L\{1\}(p) = \int_o^{\propto} i.e^{-pt}dt - \underset{m \to \propto}{Lim} \int_o^m e^{-pt}dt$$

$$= \underset{m \to \infty}{Lim} \left[\frac{e^{-pm}}{-p} + \frac{1}{p} \right] = 1/p \tag{5.3}$$

Where m is an arbitrarily large constant. In a similar way, we have

$$L\{t\}(p) = \int_o^{\propto} te^{pt}dt = \underset{m \to \infty}{Lim} \int_o^m te^{-pt}dt$$

$$= \underset{m \to \infty}{Lim} \left\{ \left[\frac{te^{-pt}}{-p} \int_o^m + \int_o^m \frac{e^{pt}}{p} dt \right) \right\} \tag{5.4}$$

$$= \frac{1}{p^2}$$

In the same vein, by induction, we can show that:

$$L\{t\}(p) = 1/p, L\{t\}(p) = \frac{1}{p^2}, L\{t^2\}(p) = \frac{2!}{p^3}, \ldots$$
$$= L\{t^n\}(p) = \frac{n!}{p^{n+1}}.$$

We can also obtain the Laplace transform for exponential, cosine, and sine functions in the following ways:

$$L\{e^{i\alpha}\}(p) = \int_0^\infty e^{-pt} e^{i\alpha t} dt = \frac{1}{i\alpha - p} \lim_{N\to\infty} e^{(i\alpha - p)}\Big|_0^N$$
$$= \frac{1}{p - i\alpha} = \frac{p + i\alpha}{\alpha^2 + p^2} \quad \textbf{(5.5)}$$

Thus, this implies that:

$$L\{e^{i\propto}\}(p) = L\{Cos\alpha t + iSin\alpha t\}(p)$$
$$= \frac{p}{p^2 + \alpha^2} + i\frac{\alpha}{p^2 + \alpha^2} \quad \textbf{(5.6)}$$

i.e.

$$L\{Cos\alpha t\} = \frac{p}{p^2 + \alpha^2} \quad \textbf{(5.7)}$$

$$L\{\sin\alpha t\} = \frac{\alpha}{p^2 + \alpha^2} \quad \textbf{(5.8)}$$

THEOREM 5.1

Let the Laplace of $f(t)$ be $F(p)$. Then

1. $L\{f(kt)\}(p) = kF(\frac{p}{k})$, for some constant $k > 0$
2. $L\{f(t-\tau)\}(p) = e^{2p\tau}F(p)$ for some $\tau > o$
3. $L\{e^{\alpha t} f(t)\}(p) = F(p-\alpha)$ Then shift theorem

CHAPTER 6

Vector Spaces and D' Operator Method

Abstract: Linear space structures through vector space and inner products are examined. The D' operator method, Cauchy-Schwartz inequality, is used. 'The D' operator is applied to obtain solutions to some practical problems.

Keywords: 'D' operator method and Cauchy-Schwartz inequality are treated, Linear space structures, Vector space and inner products.

INTRODUCTION

This chapter, the theory of linear space through vector spaces will be considered together with inner product structures and other related basic concepts in functional analysis. We discuss topics like linear independence, linear transformations, linear operators, and linear functionals. Differential ('D') operator's method will be used to solve a variety of problems which include LC, LRC, Comet- Helle, and elliptical movement of the Earth with the satellite movement or the trajectory plotted using Maple software.

Vector Spaces

Let us examine the vector spaces over field (real or complex) numbers. In this chapter, we will show that the solutions of differential equations existing in some interval of interest constitute a vector space over the field.

Definition 6.1.

A mapping $f : X \to X$ on set X is called an **internal** composition on it, while the mapping $f : X \times Y \to X$ defined on a non-void set X, Y is called an **external** composition on X on Y.

Vector Spaces

Let $(F, +,.)$ be a field. Then a non-empty set V is called a vector space over the field F if in V, there is a defined internal ($\oplus$) and external binary composition $\circledast$ such that the following conditions are satisfied([1,3-4]):

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$$\left.\begin{array}{l} V_1: \ (V,\oplus)\, is\, an\, abelian\, group + \\ V_2: \ \alpha*(x\oplus y) \ = \ \alpha*x \oplus \alpha*y \\ V_3: \ (\alpha+\beta)*x \ = \ \alpha*x + \beta*x \\ V_4: \ (\alpha.\beta)*x \ = \ \alpha*(\beta*x) \\ V_5: \ \ 1*x \ = \ \ x \end{array}\right\} \quad \mathbf{(6.1)}$$

For all α,β in the set of real or complex numbers $\mathcal{R}\ or\ \mathfrak{C}$; $x, y \in V$ and F is the multiplicative identity of the field.+ Abelian groups are groups that are commutative, that is, properties V_2 to V_5 are satisfied. In a simple language of vector spaces, the condition V_1 is about the operation being closed under the operations of addition and multiplication ([8]).

Example 6.1

Following are vector spaces $V(F)$ over the field F :

1. The set of n-tuples over the real (Euclidean space, $\Re^n$) or complex field (unitary space, $\mathfrak{C}^n$) where the operation of addition and scalar multiplication are expressed as follows:

$$ax + y = a(x_1, x_2, ..., x_n) + (y_1, y_2, y_{3,...,}y_n)$$
$$= (ax_1, ax_2, ..., ax_n) + (y_1, y_2, y_{3,...,}y_n)$$
$$ax = a(x_1, x_2, ..., x_n) = (ax_1, ax_2, ..., ax_n)$$
$$\forall x, y \in V(\Re^n\ or\ \mathfrak{C}^n\ ), a \in F(\Re^n\ or\ \mathfrak{C}^n\ )$$

2. The set of polynomials $P = \{p_n(x)\}$, where addition is defined as the addition of polynomials and scalar multiplication defined as the product of a polynomial and a scalar $a \in F(\Re^n\ or\ \mathfrak{C}^n\ )$. The set of $m\times m$ matrices in which the elements are real numbers is a vector space over the field of real or complex numbers. We denote the operation on $n\times m$ matrices by the usual matrix operation:

i.e.

$$\mathrm{V}=\{[\mathrm{a}_{\mathrm{i,j}}]_{\mathrm{n}\times m};\mathrm{a}_{\mathrm{i,j}} \in \Re\} \quad \mathbf{(6.2)}$$

$$A+B=[a_{i,j}]_{n\times m}+[b_{i,j}]_{n\times m}=[a_{i,j}+b_{i,j}]_{n\times m}, a_{i,j}, b_{i,j}\in\Re$$
$$\alpha A=\alpha[a_{i,j}]_{n\times m}=[\alpha a_{i,j}]_{n\times m}$$

3. The space of continuous function $C(\Re^n \ or \ \mathfrak{C}^n)$ is defined as the field $F(\Re^n \ or \ \mathfrak{C}^n)$ with the usual addition and scalar multiplication defined by:

$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ (\alpha f)(x) &= \alpha f(x)\end{aligned} \qquad \textbf{(6.3)}$$

$$\forall f, g \in C(\Re^n \ or \ \mathfrak{C}^n), \forall a \in F(\Re^n \ or \ \mathfrak{C}^n)$$

Problem 6.1

1. Show that the following sets are vectors spaces over the indicated field and have the natural definition for addition and scalar multiplication:

1. $$V = \left\{\alpha_1 e^x + \alpha_2 e^{2x} \middle| \alpha_1, \alpha_2 \in \Re\right\}$$

2. $$V = \{(\alpha_1, \alpha_2, \alpha_3): \alpha_1, \alpha_2, \alpha_3 \in Q, \alpha_1 = 2\alpha_2, \alpha_2 = 3\alpha_3\}$$

where Q is the set of rational numbers.

Vector Sub-Spaces

Let $V(F)$ be a vector space over the field F, then the subset $W(F)$ of $V(F)$ is said to be the vector subspace of $V(F)$, if it is itself a vector space under the induced operation defined on it. Induced operation simply means the same operation on W inherited from the parent vector space V (F).

Theorem 6.1

A subset of a vector space $V(F)$ is a subspace of it if and only if:

(i) $W \neq \phi$, (ii) $\alpha x, x-y \in W$ for $x, y \in W, \alpha \in F$.

The conditions αx and $x-y$ are often referred to as closure under the operations of multiplication and addition, respectively.

Throughout this chapter and preceding ones, we would understand closure in this context except stated otherwise.

CHAPTER 7

Solutions of Differential Equations by Power Series

Abstract: In this Chapter, the power series method for generating linearly independent solutions to ordinary differential equations is considered. The method is applied to the Bessel, Hypergeometric, Legendre and Airy equations. Some special topics for transforming nonlinear equations to linear ones by the change of variables are considered, including corresponding Maple examples for obtaining symbolic and numeric solutions to ordinary differential equations using power series and other special functions.

Keywords: Airy equations, Bessel, Hypergeometric, Linearly independent solutions, Legendre equations, Numeric solutions, Maple software, Power series method, Special functions, Symbolic solutions.

Power Series Solution

Certain classes of differential equations are not easily solvable using traditional techniques so far enumerated in the previous chapters. In this condition, we are forced to seek more advanced techniques like the power series method to find solutions to problems [1, 3, 6-7]. Power series method, though cumbrous, provides a technique to generate linearly independent solutions to many differential equations. The major problem is how to ensure that the series solution convergences in some neighborhood of interest. The other problem is how to obtain a solution when the problem contains singularities.

The following definitions are useful for the treatment of power series:

Definition 7.1

A function $f(z)$ is said to be analytic at a point $z = z_0$, if it is not only differentiable at z_0 but also on every neighborhood of z_0. The point z_0 where $f(z)$ analytic is known, in complex analysis, as a singular point or simply a point of singularity [2, 4].

Classification of Singularities

The singularity of $f(z)$ at a point $z = z_0$ in the complex plane can be classified as being poles, essential, or removable singularity. At times, the classification is traditionally made through of a series called Laurent Series, in which case, the type

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of singularity involved is identified from the principal part of the series. At other times, classifications are made from some classical definitions. Here in the chapter, we will consider various kinds of singularities using the two elucidated approaches.

Power Series

A function $f(z)$ analytic in a domain could be said to have a power series expansion if it has a unique representation:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n \tag{7.1}$$

in the region $D = \{ z : |z - z_0| < r \}$.

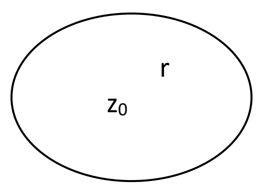

Fig. (7.1). Circle about z_0 ,of radius r .

The region D in Fig. (**7.1**) is called the region of convergence (Circle of convergence) of the series in region D. r is referred to as the radius of convergence of the power series.

The radius of convergence r is determined from the following criteria:

$$\begin{aligned} \frac{1}{r} &= \lim\sup \sqrt[n]{|a_n|} \\ &= \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \end{aligned}$$

or

$$\frac{1}{r} = \lim_{n\to\infty} \left(\sup \sqrt[n]{|a_n|} \right)$$

where a_n is defined in equation. (7.1) and r is the radius of convergence of the power series.

Taylor Series

An analytic function $f(z)$, which is defined in a domain D is said to have a Taylor series expansion; if it is expressible in a power series form and converges within the circle of convergence of radius r.

Thus:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n \qquad \textbf{(7.2)}$$

for $|z - z_0| < r$.

The radius of convergence is the shortest distance between z and z_0.

Some complex functions are analytic in an annular (ring shaped region). The power series for this function is readily derived from the Laurent series. Laurent series is named after N.M. Laurent (1983 – 1984), a French Mathematician.

Laurent's Series

Let $f(z)$ be analytic in an annual region ($r_2 < z < r_1$). Then $f(z)$ can be put in a unique form as (see [2, 4]):

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=0}^{\infty} a_{-n} (z - z_0)^{-n} \qquad \textbf{(7.3)}$$

where the coefficient a_n is $a_n = \dfrac{1}{2\pi} \displaystyle\int_{R_1} \frac{f(z)dz}{(z - z_0)^{n+1}}$

and

$$a_{-n} = \frac{1}{2\pi} \int_{R_2} \frac{f(z)dz}{(z - z_0)^{-(n+1)}} \qquad \textbf{(7.4)}$$

APPENDIX A

Brief Highlights about Maple Software

Abstract: Many real-life problems can be solved through modeling and simulation and Maple 2022 is the world-leading software used by mathematicians, physicists, economists, engineers, and educators for the problem solving task. The power of Maple and the MapleSim software are exploited in this section. We present the starting process with the software and demonstrate the application of the software *via* some selected problems.

Keywords: 2D, 3D plots, Animation, C Codes, Hybrid computations, Maple, MapleSim, MapleSim, Monte Carlo Simulation, Numerical, Symbolic, Simulation.

A.1. POWER OF MAPLE

Maple has the most powerful Math engine, and smart document interface, along with Maple add-in and grid computing facilities for symbolic, numerical, and hybrid computation, sophisticated 2D, 3D plotting and animation, and document and word processing tools.

Furthermore, Maple T.A (Test and Assessment) has an E-learning solution. Maple T A is an easy-to-use web-based system for creating tests and assignments and automatically assessing students' responses and performance. It has Maple T.A. placement Test suite to deliver tests online which reduces the cost of administration and marking examinations using paper type.

A.1.1 Maple Net

- Maple Net: A facility that allows easy sharing of Maple documents, calculator and technical application. There is also MapleSim 2022 for the simulation of engineering and real-life processes.

A.1.2 Calculus Kits

- Calculus Kits are for students and teachers to interact with each other while solving mathematical problems.

A.1.3 Users

- Maple is software that can be used by mathematicians, physicists, engineers, chemists, social scientists and educators.

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1.4 Maple Portal

- Maple makes use of what is called Portals. The Maple Portal is designed as a starting place for any Maple user. There are 3 types of portals in Maple that are related to our study in this textbook and these are:
- Portal for Engineers: which contains tools used by Engineers in solving mathematical problems. Engineering packages contain a dynamical system toolbox, scientific constants, scientific error analysis, tolerance and units.
- Portal for Students: Student packages are available for the following topics: Pre-calculus, calculus, vector calculus, differential equations, linear Algebra, and multivariate calculus.
- Portal for Math Educators: This portal contains information and tools for education, assessment, Maple Test and assessment of students. This portal contains student packages that allow instructors to deliver the course contents effectively; give students insight into understanding basic mathematical concepts and enhance their problem-solving practical skills. There is also a survival kit to enhance students' mathematical mastery of topics in the portal for students

A.1.5 Help Resources and Maple Tour

Maple also has Help Resources and Maple Tour to give tutorials on how to use the resources in Maple and Help system to help the users out of perceived problems and many examples on how to use maple resources. There is also a Quick reference card. This card gives vital information on how to make use of resources like the type of modes for creating documents in Maple. It also gives information on Toggle Math/Text entry mode, how to evaluate math expressions and display results in line; common operations available in the Maple in both document and worksheet Modes; 2-D math editing operations, keyboard shortcuts, and operations plotting and animation.

A.1.6 User Manuals and Web links

User Manuals: This manual gives comprehensive information about Maple, tutorials, and examples on Maple. The manual contains how to get started with maple toolboxes, the user manual and the programming guild.

Web links: This is the hyperlink to Maple soft Company, which is the developer and marketer of Maple software. The links provide information and registration of the company, and show how to register and take part on webinars, an online seminar series. It also provides information on how to get online resources on Maple.

A.2 Getting Started

A.2.1 Maple Tutorial

Maple tutorial helps to get started with the software, learn about the tools available in Maple, and lead you through a series of problems. It guides you on how to enter simple expression, functions, matrices, complex numbers, and evaluate expression and plotting functions.

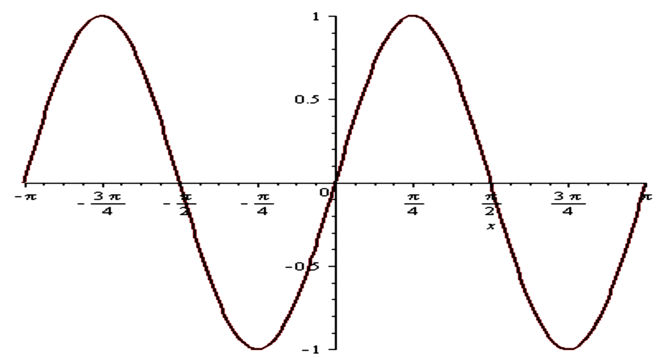
Maple has so many interesting modelling and simulation facilities as we are not going to make a discussion on them but we will demonstrate their applications in Maple Examples.

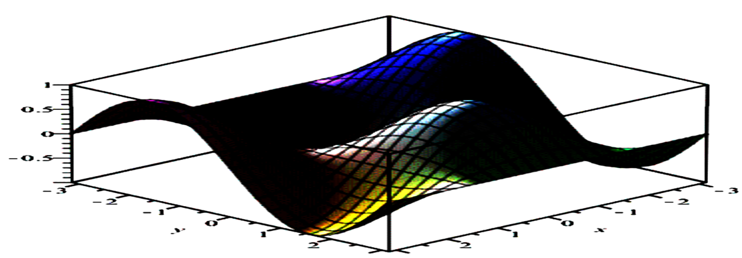
Examples on Graphs and Animations

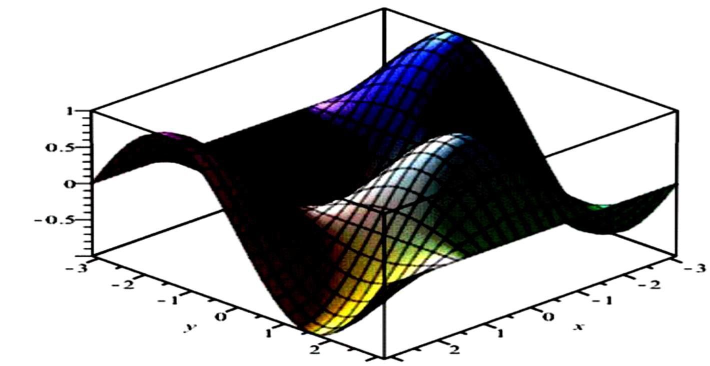
Example A1

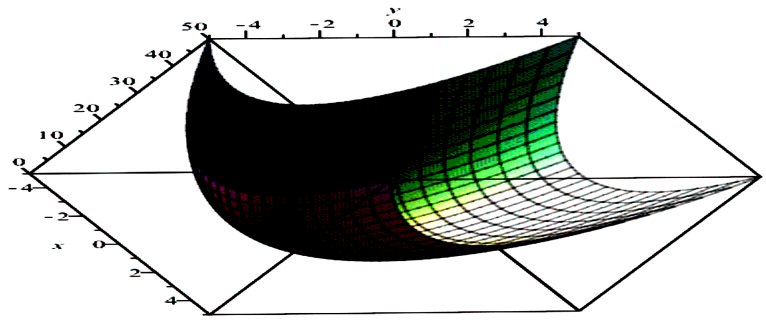
To plot the graph of sine function in the worksheet mode, type in the command:

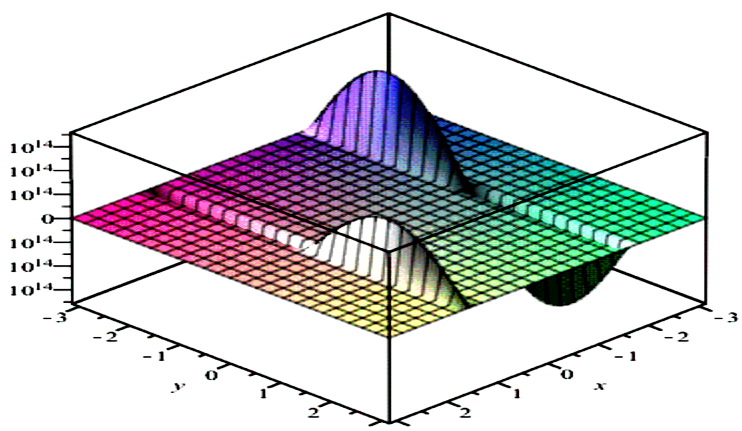
> Plot (sin (2*x),x =-Pi..Pi, thickness=2);

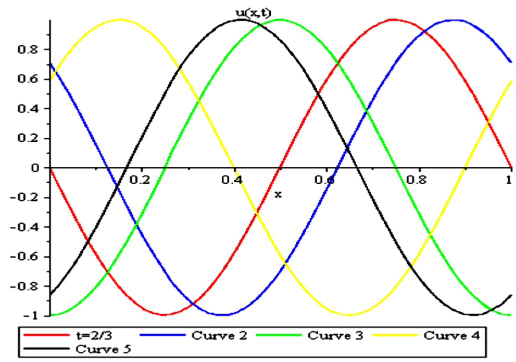


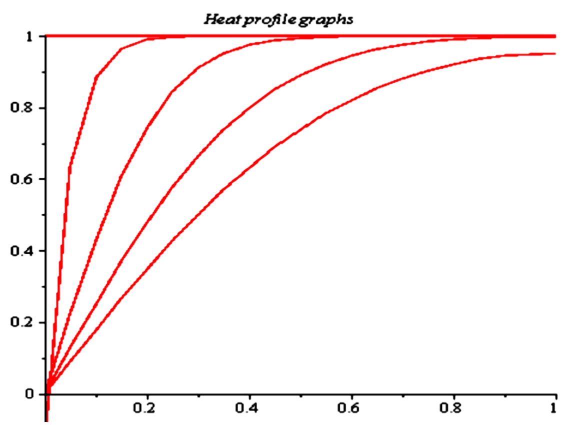



1
0.5
0
-0.5
-3
-2
-1
y
0
1
2
-3
-2
-1
0
x
1
2

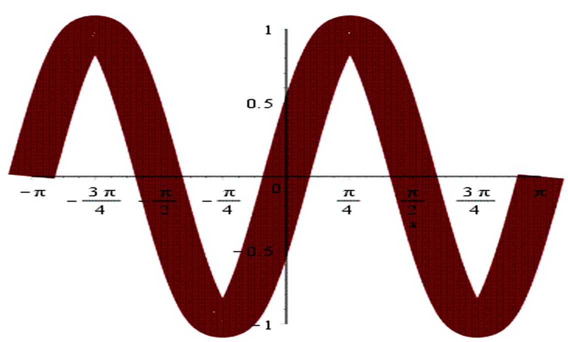

y
50
-4
-2
0
2
4
40
30
20
10
0
-4
-2
x
0
2
4


0
-3
-2
-1
y
0
1
2
-3
-2
-1
0
x
1
2





$plot(\sin(2 \cdot x), x = -\text{Pi}..\text{Pi}, thickness = 30);$

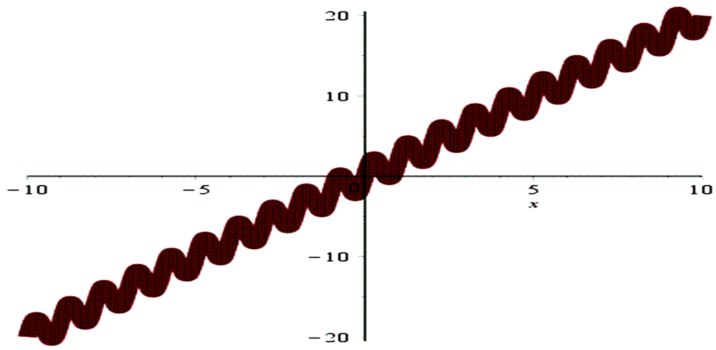


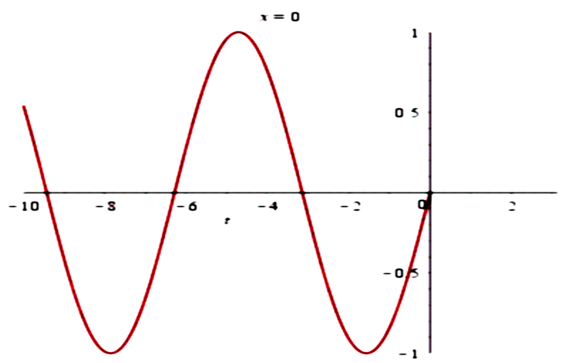
$> f := 2 \cdot \sin(2 \cdot \text{Pi} \cdot x) + 2 \cdot x;$

$$f := 2\sin(2\pi x) + 2x$$

$> plot(f, x = -10..10, thickness = 10);$

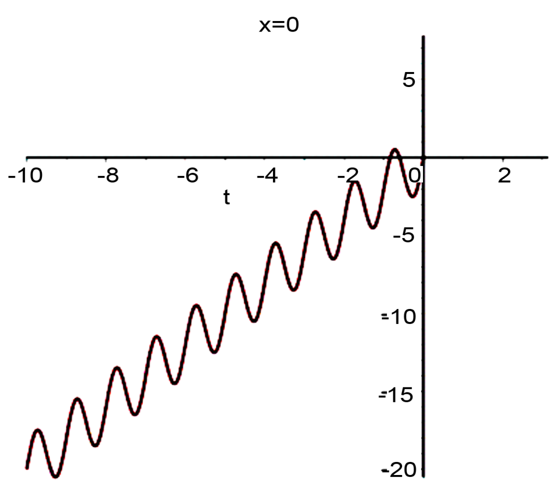
> >*animate*(*plot*, [sin(*t*), *t* = −10 ..*x*], *x* = 0 ..Pi);





>

\> *animate*(*plot*, [2·sin(2· Pi·*t*) + 2·t, t = −10 ..x], *x* = 0 ..Pi);

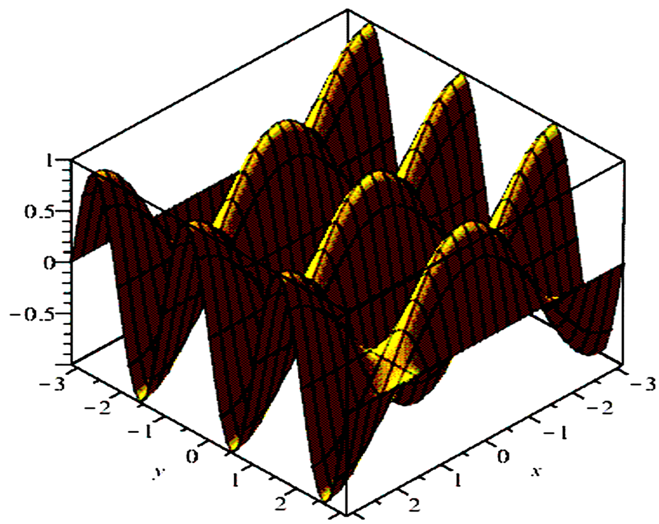


> $restart$

> $with(plots)$:

> $animate3d(\cos(t \cdot x) \cdot \sin(3 \cdot t \cdot y), x = -\mathrm{Pi}..\mathrm{Pi}, y = -\mathrm{Pi}..\mathrm{Pi}, t = 1..2);$

We can extend the plot to 3D using document mode: type in the following two dimension function w=w(x, y) and highlight the equation, right click the 3D plot, we have:



>

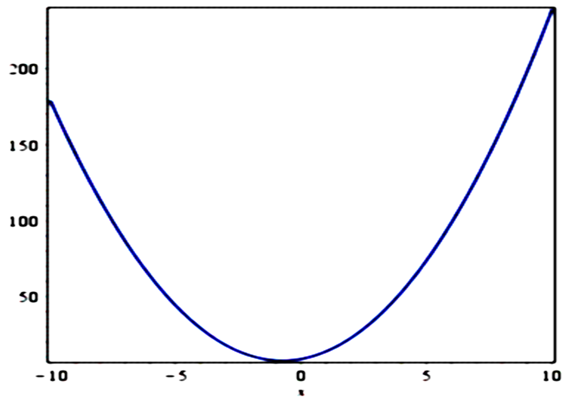
> $restart$

> $with(plots)$:

> $with(DEtools)$:

In the document mode, type the equation and highlight it and right click to select the 2D plot, then we have the plot:

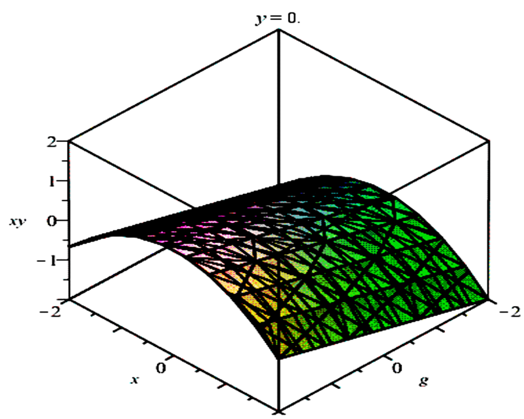
$y = 2x^2 + 3x + 9 \rightarrow$



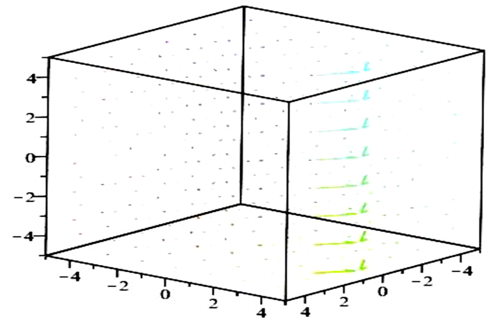
>

>We can also replicate the above plot using worksheet mode by typing in the equation and right- click and select 3D plot. You can also use the plot builder to have a variety of 3D-plots and even animate the plots too.

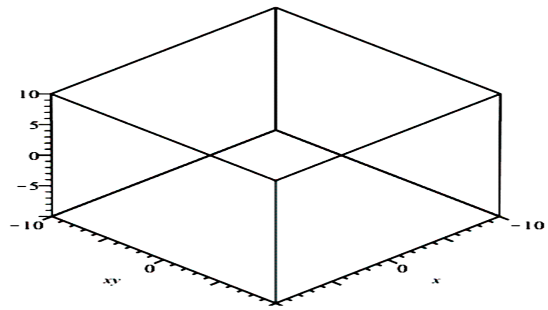
$$w = x^2 + y^2 + 3\,xy + \frac{2\,xy}{x^2 + \sin(xy)} \rightarrow$$



>**w=3x^2** $+ 3\,xy + y^2 + \dfrac{2\,xy}{x^2 + \sin(xy)}$;



>

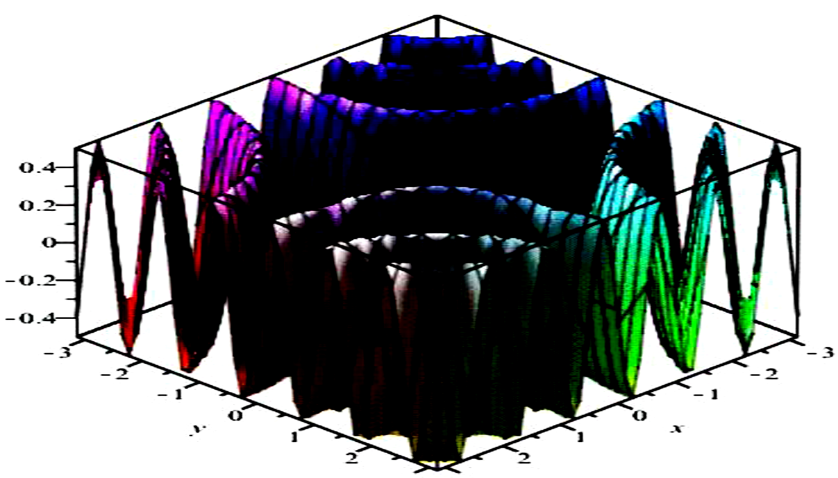


>

Using worksheet mode, you can animate a plot using the command with (plots) together with animate3d. For example, type in:

>*with*(*plots*) :

animate3d($\sin(t \cdot x \cdot y) \cdot \cos(t \cdot x \cdot y)$, $x = -\text{Pi}..\text{Pi}$, $y = -\text{Pi}..\text{Pi}$, $t = 0..1$)

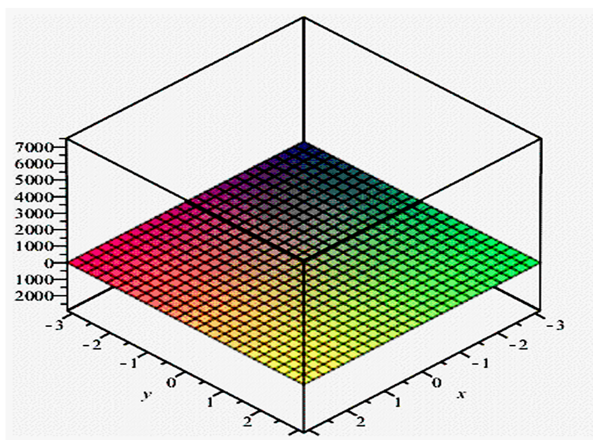


Highlight the 3D-plot and select the type of animation, whether short-time animation or continuous one. In Maple software, the memory can be cleared using 'restart'.

restart

with(*plots*) :

animate3d($\exp(t\cdot x\cdot y)\cdot\sin(t\cdot x\cdot y)\cdot\cos(t\cdot x\cdot y), x=-\mathrm{Pi}..\mathrm{Pi}, y=-\mathrm{Pi}..\mathrm{Pi}, t=0..1$)



Maple contains several facilities for computation using Linear Algebra. Type in with (LineraAlgebra) with 'semicolon' to display the linear algebra facilities in the maple software. We can suppress this by using colon as usual.

> *with*(*LinearAlgebra*);

In the worksheet mode, a vector and a matrix can be typed in as follows:

x=Vector ([1, 0,-2, 3);

$$x := \begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix}$$

[*&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*]

>A:=Matrix([[1, 2, 0, 3], [0, 0, -1, 4], [0, 0, -3, 2], [2, 1, 0, 2]]);

$$A := \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & -3 & 2 \\ 2 & 1 & 0 & 2 \end{bmatrix}$$

The element on the first row fourth colon can be displayed by typing in:

> $A[1, 4]$;

3

> $A[3, 3]$;

-3

A matrix A can be multiplied by itself using the code:

> $A.A$;

$$\begin{bmatrix} 7 & 5 & -2 & 17 \\ 8 & 4 & 3 & 6 \\ 4 & 2 & 9 & -2 \\ 6 & 6 & -1 & 14 \end{bmatrix}$$

Matrix A can be post multiplied using the vector x as follows:

> ***A.x***

$$\begin{bmatrix} 10 \\ 14 \\ 12 \\ 8 \end{bmatrix}$$

B: =Matrix ([[**1,2],[5,7],[3,5], [0,3]]);**

$$B := \begin{bmatrix} 1 & 2 \\ 5 & 7 \\ 3 & 5 \\ 0 & 3 \end{bmatrix}$$

> ***A.B;***

$$\begin{bmatrix} 11 & 25 \\ -3 & 7 \\ -9 & -9 \\ 7 & 17 \end{bmatrix}$$

> ***B.A;***

Error, (in LinearAlgebra:-Multiply) first matrix column dimension (2) <> second matrix row dimension (4)

> $h := i \rightarrow i^2;$

$$h := i \rightarrow i^2$$

> **y := Vector(8, h);**

$$y := \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \\ 25 \\ 36 \\ 49 \\ 64 \end{bmatrix}$$

> *# generate the HilbertMatrix*

> **H:= Matrix(5,5, (i,j) -> 1/(i+j-1));**

$$H := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{bmatrix}$$

> $C := \langle\langle 1, 2, 3\rangle|\langle 0, 0, 1\rangle|\langle 0, 0, 1\rangle\rangle;$

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

> *# Find the basis for A and C;*

> $NullSpace(A);$

$\{\ \}$

> $NullSpace(C);$

$$\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

> *# A has no basis;*

> $d := \langle 1, 2, 0, -1\rangle;$

$$d := \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

>z:=LinearSolve(A,d);

Warning, inserted missing semicolon at end of statement

$$z := \begin{bmatrix} -\frac{6}{5} \\ \frac{1}{5} \\ \frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$$

> E:=IdentityMatrix(4);

Warning, inserted missing semicolon at end of statement

$$E := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

> $Determinant(A);$

-30

> $Rank(A);$

4

> $l2ip := (f, g) \rightarrow int(f(x) \cdot g(x), x = 0..1);$

$$l2ip := (f, g) \rightarrow \int_0^1 f(x)\, g(x)\, \mathrm{d}x$$

>N:=f ->sqrt(%(f,f));

$$N := f \rightarrow \sqrt{\%(f, f)}$$

> $unassign('x');$

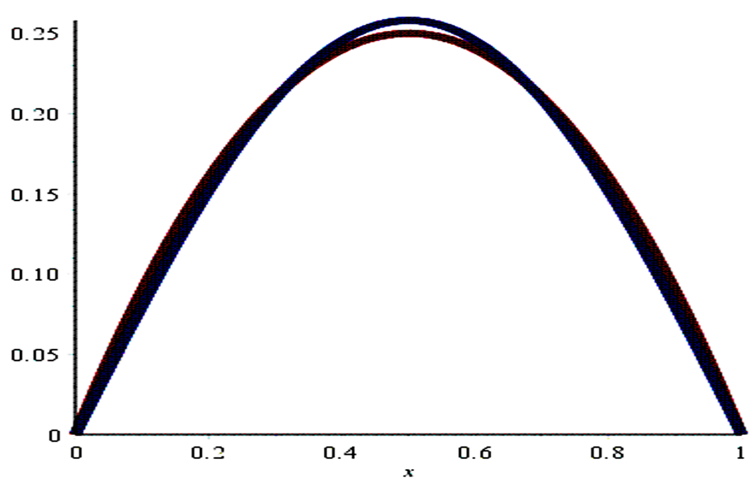
> $f := x \rightarrow x \cdot (1 - x);$

$$f := x \rightarrow x\,(1 - x)$$

> $g := x \rightarrow \frac{8}{\pi^3} \cdot \sin(\mathrm{Pi} \cdot x);$

$$g := x \rightarrow \frac{8 \sin(\pi\, x)}{\pi^3}$$

> plot({f(x),g(x)},x = 0 .. 1,thickness = 6);



>>> $edn := diff(y(x), x) = -\frac{x}{y};$

$edn := \frac{d}{dx} y(x) = -\frac{x}{y}$

> $with(plots)$

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

> $edn := diff(y(x), x) = -\frac{x}{y};$

$edn := \frac{d}{dx} y(x) = -\frac{x}{y}$

> $c := gradplot\left(-\frac{x}{y}, \mathrm{x} = -10..10, \mathrm{y} = -10..10\right)$

$c := PLOT(...)$

>

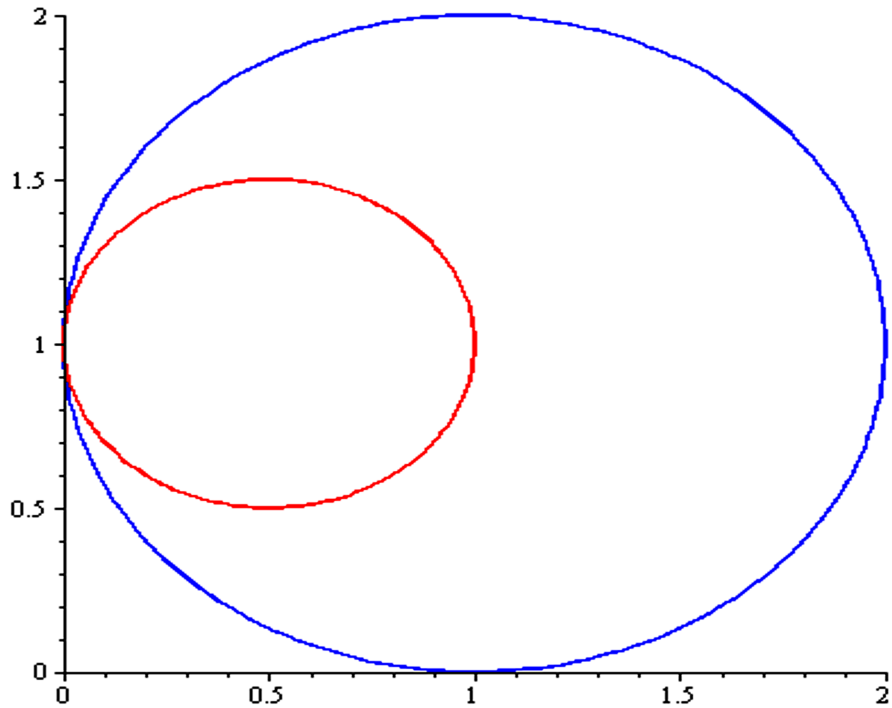
> with(plottools):

> with(plots):

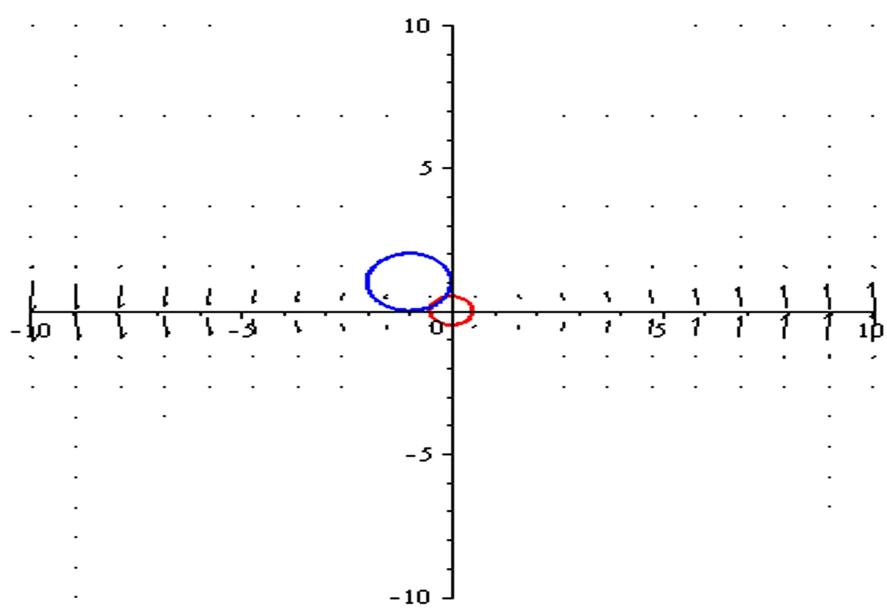
> c1:= circle([1,1], 1,color=blue):

> c2:=circle([1/2,1], 1/2,color=red):

> display([c1,c2);



> $display(c, c2, c1);$:



> $with(plots)$:

> $c := sphere([1, 1, 1], 3.3)$:

> $display(c, scaling = constrained, axes = boxed)$

> $d := sphere([1, 5, 1], 2)$:

> $display(c, d, scaling = constrained, style = patchnogrid)$

> $esphere([1, 5, 1], 2)$:

> $display(c, e, scaling = constrained, style = patchnogrid)$

>

> ;=

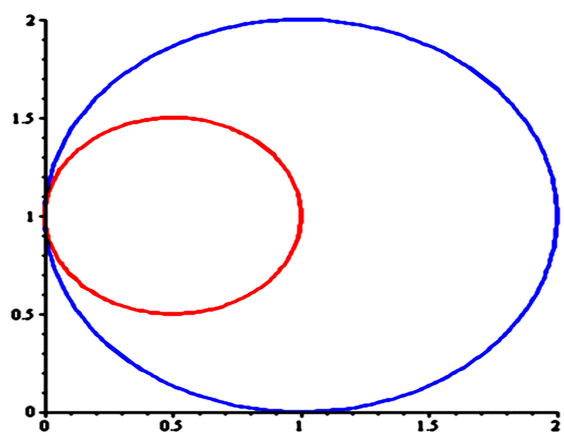
> **with(plottools):**

> **with(plots):**

> **c1 := ellipse([1,1], 1, color=blue):**

> **c2 := circle([1/2,1], 1/2, color=red):**

> **display(c1,c2);**



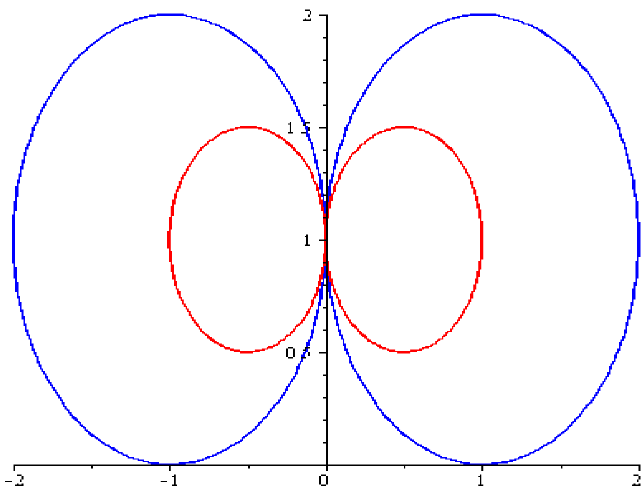
> **c3:= ellipse([-1,1], 1,color=blue):**

> c4:= circle([-1/2,1], 1/2,color=red):

> with(plottools):

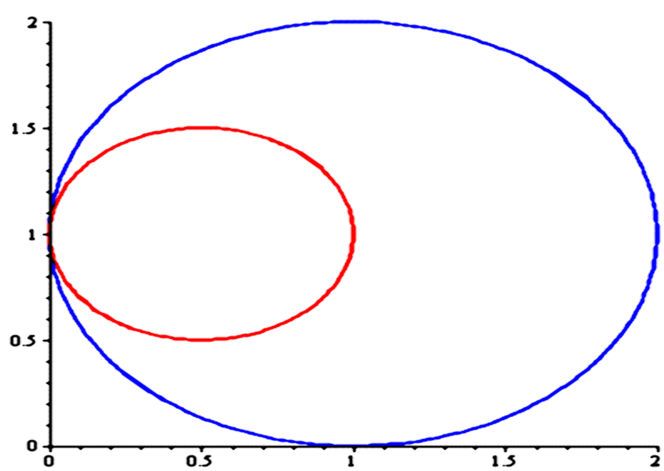
> with(plots):

> display(c1,c2,c3,c4);



> c2:=circle([1/2,1], 1/2,color=red):

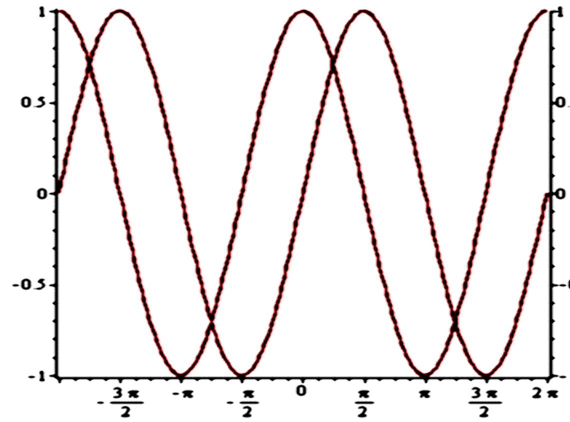
> display(c1,c2);



> *restart*

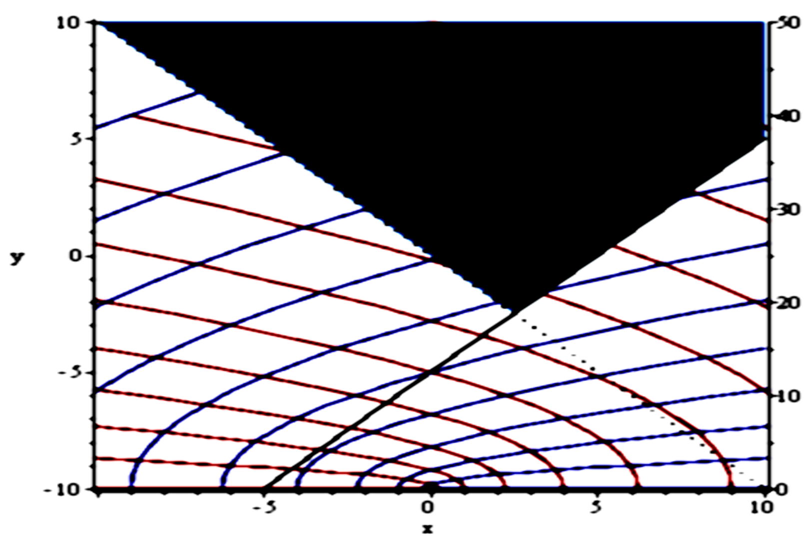
> *with*(*plots*) :

> *dualaxisplot*(*plot*(sin), *plot*(cos))



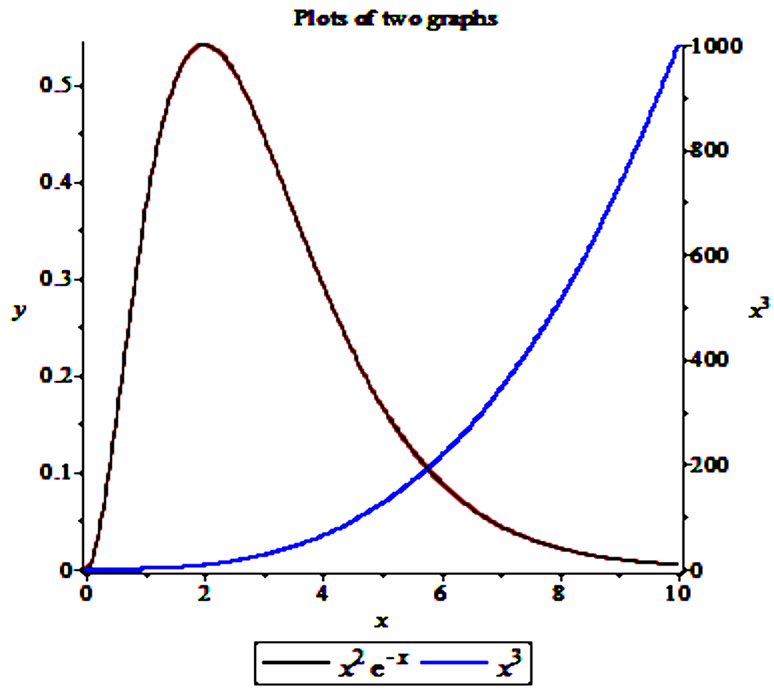
>

$dualaxisplot\big(inequal(\{x - y \le 5, 0 < x + y\}, x = -10\,..10, y = -10\,..10, optionsexcluded = (color = white)), conformal\big(z^2, z = 0\,..5 + 5\,I\big)\big)$



>
$dualaxisplot\left(plot\left(x^2 \cdot \exp(-x), x = 0\,..10, labels = [x, y], legend = x^2 \cdot \exp(-x)\right), plot\left(x^3, x = 0\,..10, color = blue, labels = \left[x, x^3\right], legend = x^3\right), title = \text{" Plots of two graphs "}\right)$

>
$dualaxisplot\left(animate\left(plot, \left[A\,x^3, color = blue, labels = \left[x, x^3\right]\right], A = 0\,..1\right), plot\left(x^2, labels = \left[x, x^2\right]\right)\right)$



> *restart*

> with(plots,[pareto]):

>pdata:= 'Engine 1'=327,

\`Engine 2\`= 240,

\`Engine 3\`=176,

\`Wire 1\`=105,

`Wire 2`=43,

`Wire 3`=36,

Oil=33,

Coils=90,

`Gear Box`=61,

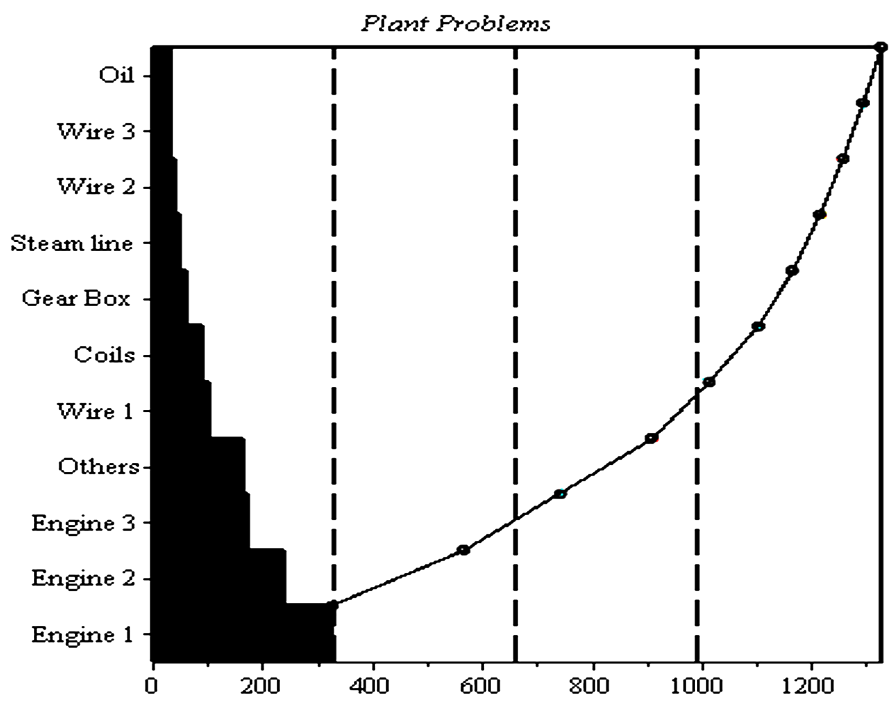
`Steam line`=50,

Others=166]:

>Fdata:=map(rhs,Pdata):

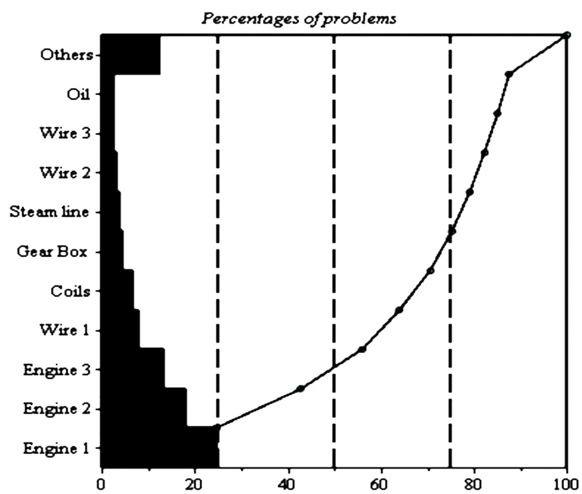
> Lab:=map(lhs,Pdata):

> > *pareto*(*Fdata*, *tags* = *Lab*, *title* = `*Plant Problems*`);



> Fdata_norm:=map((x,s) -> 100*x/s, Fdata, `+`(op(Fdata))):

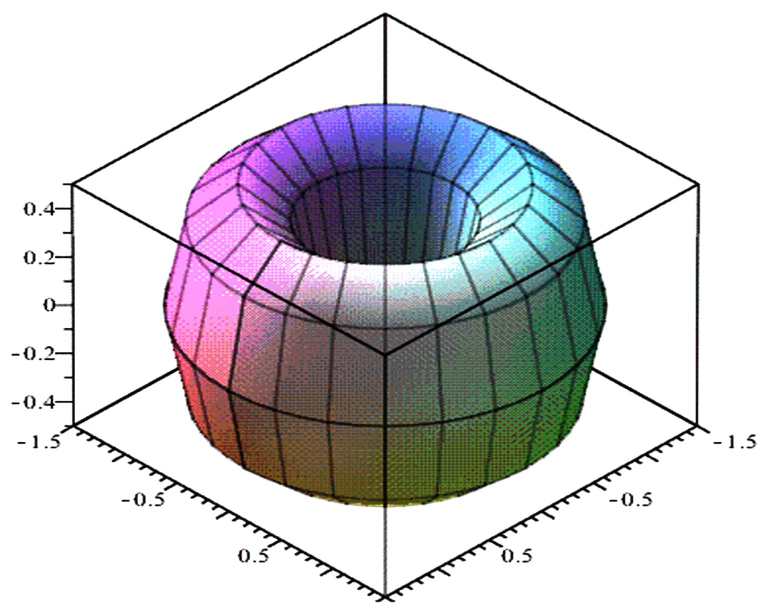
> pareto(Fdata_norm, tags=Lab, misc=Others, title=`Percentages of problems`);



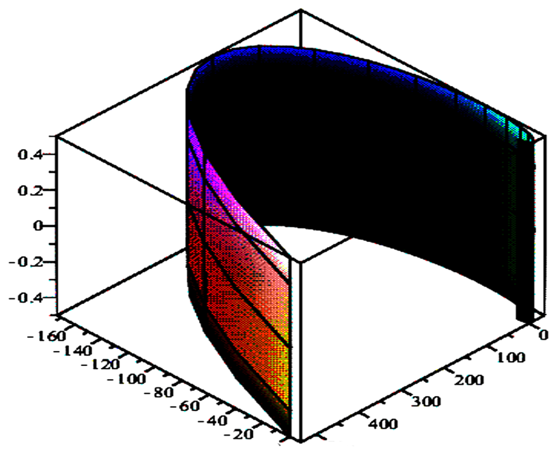
> *restart*

> *with*(*plots*) :

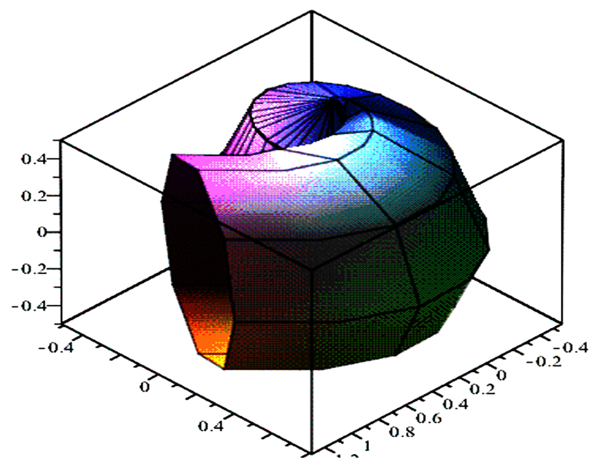
> $tubeplot([\cos(t), \sin(t), 0], t = 0 ..2\,\pi, radius = 0.5)$



> $tubeplot\left([\exp(t)\cdot\cos(t), \exp(t)\cdot\sin(t), 0], t = 0\,..2\,\pi, radius = 0.5\right)$



> $tubeplot\left([\exp(-t)\cdot\cos(t), \exp(-t)\cdot\sin(t), 0], t = 0\,..5\,\pi, radius = 0.5\right)$

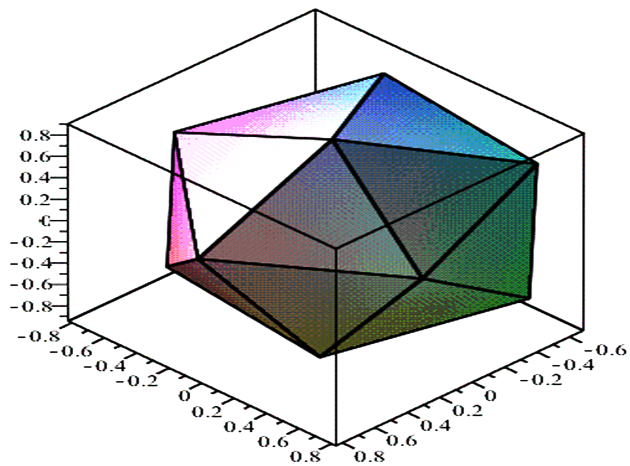


> *restart*

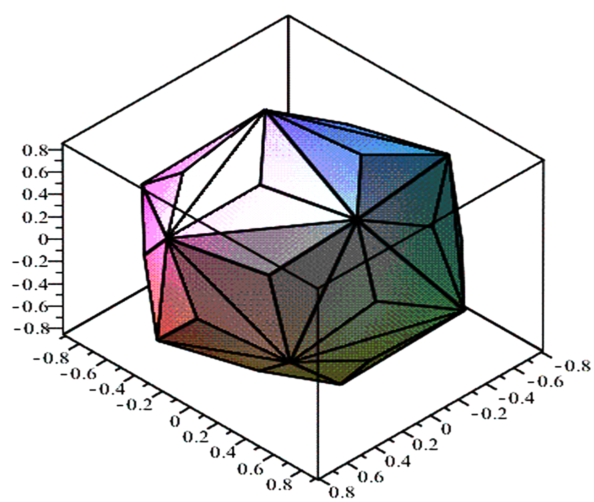
> *with*(*plots*) :

> *polyhedra_supported*( ) :

> *polyhedraplot*([0, 0, 0], *polytype* = *GyroelongatedPentagonalPyramid*, *scaling* = *constrained*)



> *polyhedraplot*([0, 0, 0], *polytype* = *TriakisIcosahedron*, *scaling* = *constrained*)



>JuliaSet:= proc(a,b)

local z1, z2, z1s, z2s,m;

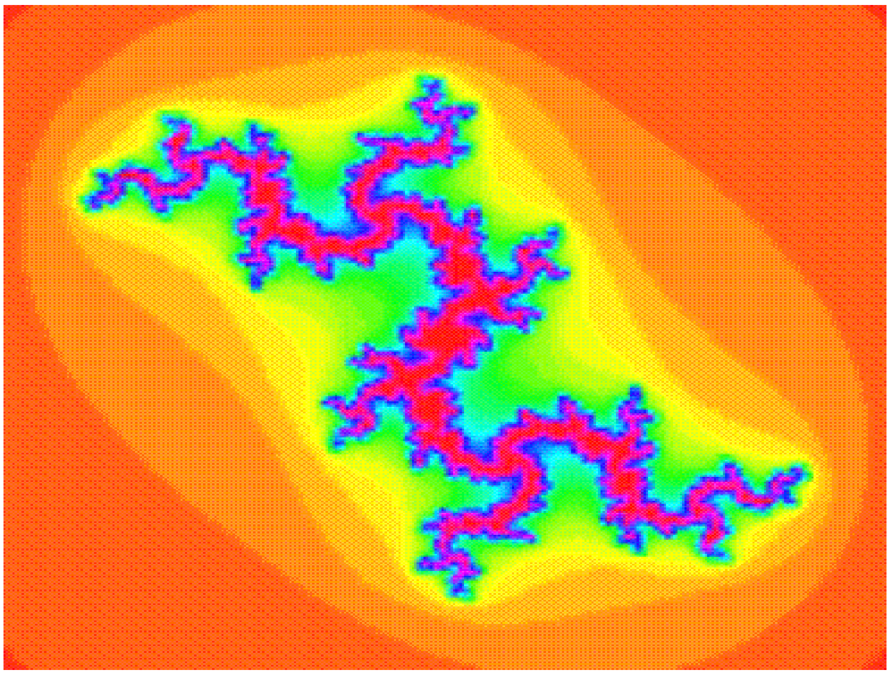
(z1, z2): = (a,b):

z1s:= z1^2:

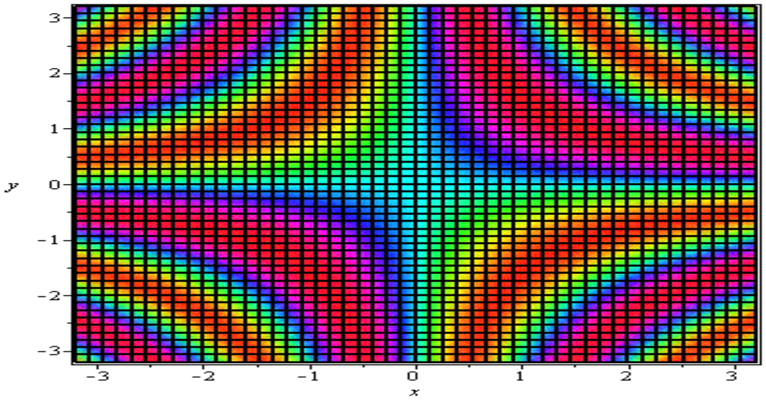
z2s: = z2^2;

```
 for m to 30 while z1s+z2s < 4 do
   (z1, z2):= (z1s-z2s, 2*z1*z2) + (0, 0.75);
   z1s:=z1^2;
   z2s:= z2^2;
 end do;
m;
end proc:
```

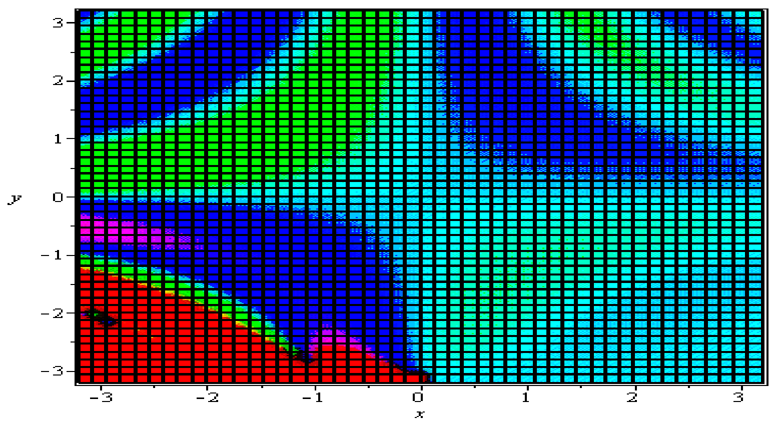
>
$densityplot(JuliaSet, -1.5..1.5, -1.4..1.4, colorstyle = HUE, grid = [150, 150], style = patchnogrid, axes = none)$



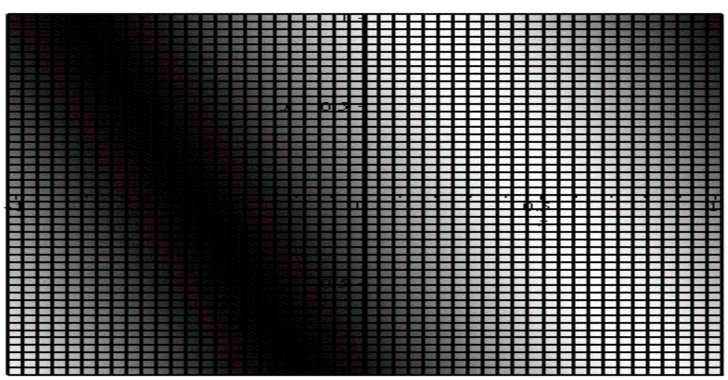
> $densityplot(\sin(x\,y), x = -\pi..\pi, y = -\pi..\pi, axes = boxed, colorstyle = HUE)$



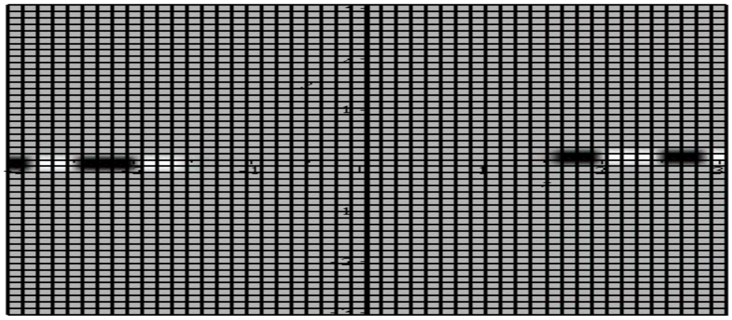
> $densityplot\left(\exp\left(-\frac{x}{y+\mathrm{Pi}}\right)\cdot\sin(x\,y), x=-\pi..\pi, y=-\pi..\pi, axes=boxed, colorstyle=HUE\right)$



> $densityplot(\sin(\mathrm{Pi}\cdot x+y), x=-1..1, y=-1..1)$



$> densityplot\left(\exp\left(\frac{x}{y}\right)\cdot\cos(\text{Pi}\cdot x - y)\cdot\sin(\text{Pi}\cdot x + y), x=-3..3, y=-3..3\right)$



$> SpaceCurve\left(\langle e^{-t}\cos(t), e^{-t}\sin(t)\rangle, t=4..8\right)$

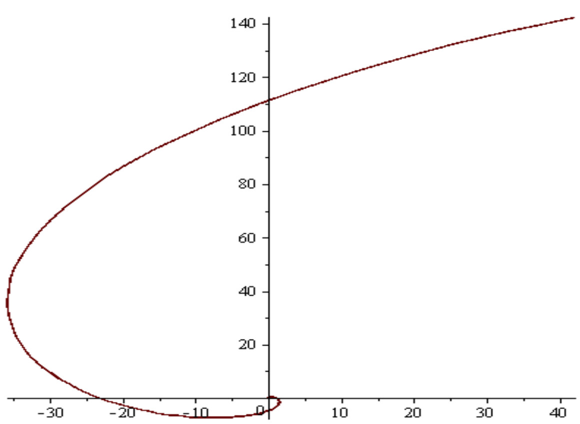
$$SpaceCurve\left(\begin{bmatrix} e^{-t}\cos(t) \\ e^{-t}\sin(t)\end{bmatrix}, t=4..8\right)$$

$> SpaceCurve(\langle\cos(t), \sin(t), t\rangle, t=1..9)$

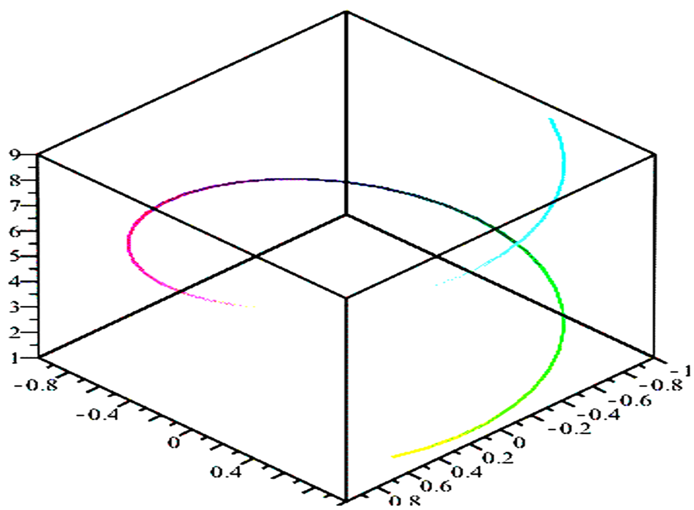
$$SpaceCurve\left(\begin{bmatrix} \cos(t) \\ \sin(t) \\ t\end{bmatrix}, t=1..9\right)$$

$> with(VectorCalculus):$

$> SpaceCurve\left(\langle e^{-t}\cos(t), e^{-t}\sin(t)\rangle, t=-5..5\right)$



> $SpaceCurve(\langle\cos(t), \sin(t), t\rangle, t = 1..9)$



N-order Nuclear Reactor Process

$with(ODETools):$

> $with(plots):$

> $eqn1 := diff(N(t), t) = \frac{(k-1)\cdot N(t)}{l} - \frac{\text{beta}\cdot N(t)}{l} + sum(\text{lambda}[i]\cdot r[i](t), i = 1..m);$

$$eqn1 := \frac{d}{dt} N(t) = \frac{(k-1)\,N(t)}{l} - \frac{\beta\,N(t)}{l} + \sum_{i=1}^{m} \lambda_i r_i(t)$$

> $eqn2 := diff(r[i](t), t) = \frac{\text{beta}\cdot N(t)}{l} - \text{lambda}[i]\cdot r[i](t);$

$$eqn2 := \frac{d}{dt} r_i(t) = \frac{\beta\,N(t)}{l} - \lambda_i r_i(t)$$

> $wih(PDETools, dchange):$

> $chngV1 := \{N(t) = sum(N[j]\cdot\exp(\text{omega}[j](t)), j = 0..m)\};$

$$chngV1 := \left\{ N(t) = \sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)} \right\}$$

> $chngV2 := \{r[i](t) = sum(r[i,j]\cdot\exp(\mathrm{omega}[j](t)), j = 0..m)\};$

$$chngV2 := \left\{ r_i(t) = \sum_{j=0}^{m} r_{i,j} \mathrm{e}^{\omega_j(t)} \right\}$$

> $P1 := op(factor(combine(expand(chngV1), power)))$

$$P1 := N(t) = \sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)}$$

> $P2 := op(factor(combine(expand(chngV2), power)))$

$$P2 := r_i(t) = \sum_{j=0}^{m} r_{i,j} \mathrm{e}^{\omega_j(t)}$$

> $evalf(\{eqn1, eqn2\});$

$$\left\{ \frac{\mathrm{d}}{\mathrm{d}t} N(t) = \frac{(k-1.)\, N(t)}{l} - \frac{1.\, \beta\, N(t)}{l} + \sum_{i=1}^{m} \lambda_i r_i(t),\ \frac{\mathrm{d}}{\mathrm{d}t} r_i(t) = \frac{\beta\, N(t)}{l} - 1.\, \lambda_i r_i(t) \right\}$$

> $subs([\mathrm{P1}, \mathrm{P2}], [eqn1, eqn2]);$

$$\left[\frac{\partial}{\partial t} \left(\sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)} \right) = \frac{(k-1) \left(\sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)} \right)}{l} - \frac{\beta \left(\sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)} \right)}{l} + \sum_{i=1}^{m} \lambda_i \left(\sum_{j=0}^{m} r_{i,j} \mathrm{e}^{\omega_j(t)} \right), \right.$$
$$\left. \frac{\partial}{\partial t} \left(\sum_{j=0}^{m} r_{i,j} \mathrm{e}^{\omega_j(t)} \right) = \frac{\beta \left(\sum_{j=0}^{m} N_j \mathrm{e}^{\omega_j(t)} \right)}{l} - \lambda_i \left(\sum_{j=0}^{m} r_{i,j} \mathrm{e}^{\omega_j(t)} \right) \right]$$

> $p := simplify(\%);$

$$p := \left[\sum_{j=0}^{m} N_j \left(\frac{d}{dt} \omega_j(t) \right) e^{\omega_j(t)} = -\frac{\beta \left(\sum_{i=0}^{m} N_i e^{\omega_i(t)} \right) - \left(\sum_{i=0}^{m} N_i e^{\omega_i(t)} \right) k - \left(\sum_{i=1}^{m} \lambda_i \left(\sum_{j=0}^{m} r_{i,j} e^{\omega_j(t)} \right) \right) l + \sum_{i=0}^{m} N_i e^{\omega_i(t)}}{l}, \sum_{j=0}^{m} r_{i,j} \left(\frac{d}{dt} \omega_j(t) \right) e^{\omega_j(t)} = -\frac{\lambda_i \left(\sum_{j=0}^{m} r_{i,j} e^{\omega_j(t)} \right) l - \beta \left(\sum_{j=0}^{m} N_j e^{\omega_j(t)} \right)}{l} \right]$$

> *map*(*simplify*, **(28)**, '*assume* = *nonnegative*')

$$\left[\sum_{j=0}^{m} N_j \left(\frac{d}{dt} \omega_j(t) \right) e^{\omega_j(t)} = -\frac{\beta \left(\sum_{i=0}^{m} N_i e^{\omega_i(t)} \right) - \left(\sum_{i=0}^{m} N_i e^{\omega_i(t)} \right) k - \left(\sum_{i=1}^{m} \lambda_i \left(\sum_{j=0}^{m} r_{i,j} e^{\omega_j(t)} \right) \right) l + \sum_{i=0}^{m} N_i e^{\omega_i(t)}}{l}, \sum_{j=0}^{m} r_{i,j} \left(\frac{d}{dt} \omega_j(t) \right) e^{\omega_j(t)} = -\frac{\lambda_i \left(\sum_{j=0}^{m} r_{i,j} e^{\omega_j(t)} \right) l - \beta \left(\sum_{j=0}^{m} N_j e^{\omega_j(t)} \right)}{l} \right]$$

Examples on image processing

> *with*(*ImageTools*) :

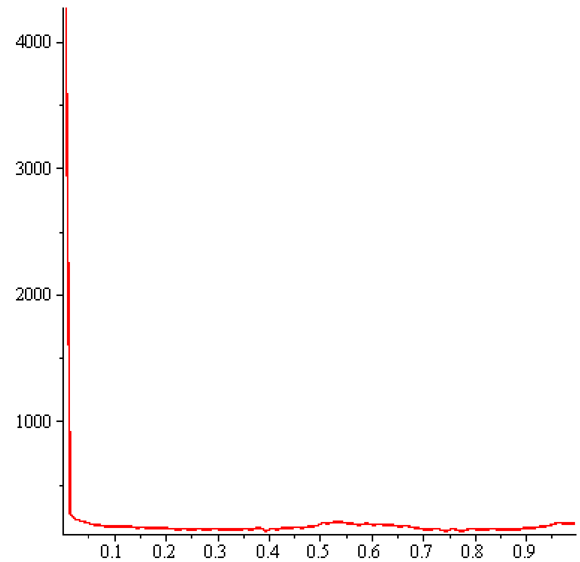
> $img1 := Create\left(100, 200, (r, c) \rightarrow evalf\left(0.5 \sin\left(\frac{r}{50} \right) + 0.5 \sin\left(\frac{c}{30} \right) \right) \right)$:

> *img2* := *Complement*(*img1*) :

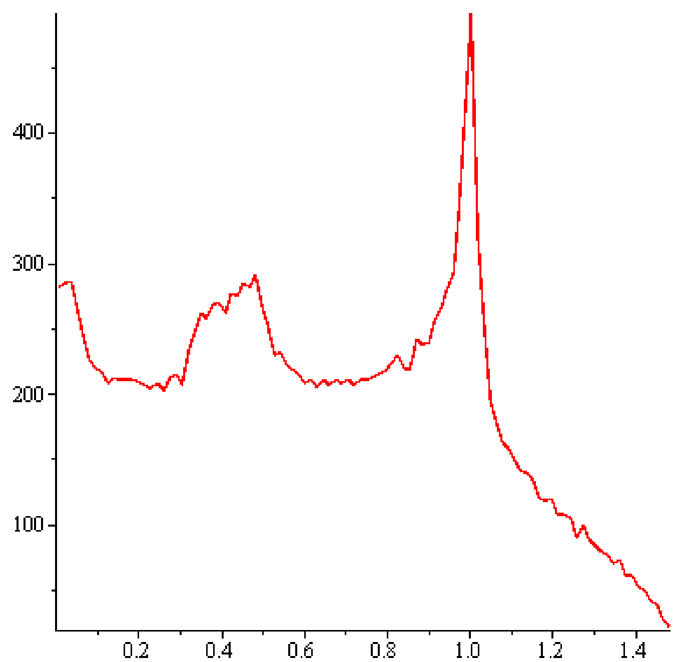
> *View*(*img1*)

> *View*([*img1*, *img2*])

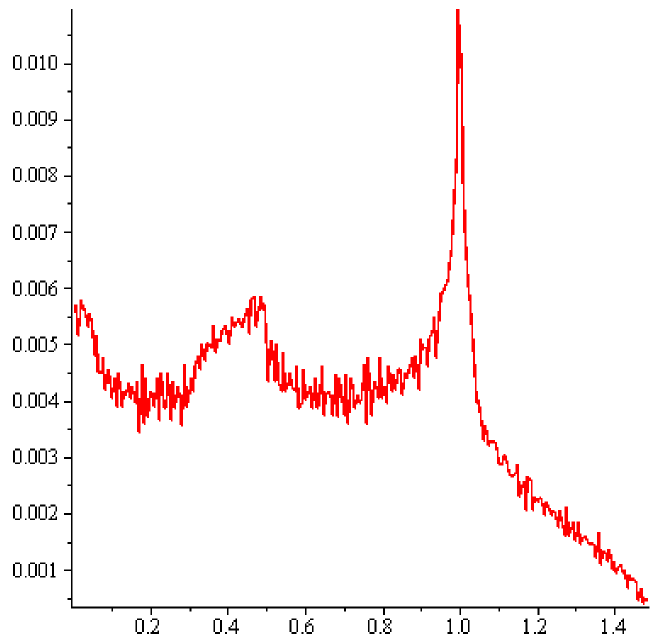
> $PlotHistogram(img1, 100)$



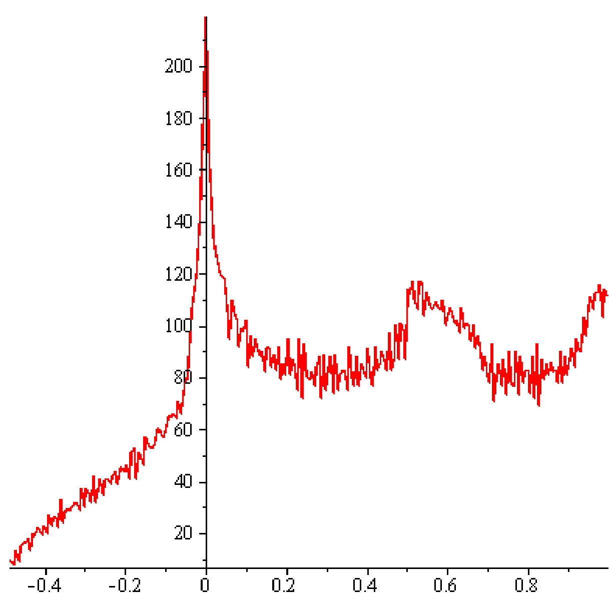
> $PlotHistogram(img2, 100, autorange)$



> *PlotHistogram*(*img2*, *autorange*, *normalized*)

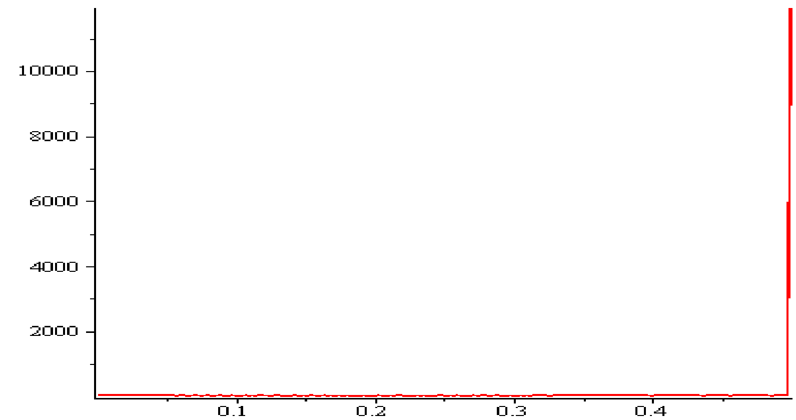


> *PlotHistogram*(*img1*, *autorange*)



> $PlotHistogram(img2, range = 0..0.5)$

Example for fitting experiments



> $restart$

> $with(Statistics):$

> $X := Vector([1, 2, 3, 4, 5, 6], datatype = float):$

> $Y := Vector([2, 5.6, 8.2, 20.5, 40.0, 95.0], datatype = float):$

> $ExponentialFit(X, Y, v)$

$1.01888654495804\, e^{0.746236510177018\, v}$

> $W := Vector([1, 1, 1, 2, 5, 5], datatype = float):$

> $ExponentialFit(X, Y, weights = W)$

$$\begin{bmatrix} 0.989482297469512 \\ 0.752656239139387 \end{bmatrix}$$

> $LinearFit([1, t, t^2], X, Y, t)$

$24.6000000000000 - 23.9892857142857\, t + 5.79642857142857\, t^2$

> $LinearFit(a + b\,t + c\,t^2, X, Y, t)$

$$24.6000000000000 - 23.9892857142857\,t + 5.79642857142857\,t^2$$

Consider now an experiment where quantities x , y and z are quantities influencing a quantity w according to an approximate relationship

$$w = a\,x + \frac{b\,x^2}{y} + c\,y\,z$$

with unknown parameters a , b , and c . Six data points are given by the following matrix, with respective columns for x , y , z , and w .

> $ExperimentalData := \langle\langle 1, 1, 1, 2, 2, 2\rangle | \langle 1, 2, 3, 1, 2, 3\rangle | \langle 1, 2, 3, 4, 5, 6\rangle | \langle 0.531, 0.341, 0.163, 0.641, 0.713, -0.040\rangle\rangle$

$$ExperimentalData := \begin{bmatrix} 1 & 1 & 1 & 0.531 \\ 1 & 2 & 2 & 0.341 \\ 1 & 3 & 3 & 0.163 \\ 2 & 1 & 4 & 0.641 \\ 2 & 2 & 5 & 0.713 \\ 2 & 3 & 6 & -0.040 \end{bmatrix}$$

> $LinearFit\left(\left[x, \frac{x^2}{y}, y\,z\right], ExperimentalData, [x, y, z]\right)$

$$0.823072918385878\,x - \frac{0.167910114211606\,x^2}{y} - 0.0758022678386438\,y\,z$$

> $NonlinearFit(a + b\,v + e^{c\,v}, X, Y, v)$

$$2.15979247107424 - 1.22391291112346\,v + e^{0.766784080984173\,v}$$

> $NonlinearFit\left(x^a + \frac{b\,x^2}{y} + c\,y\,z, ExperimentalData, [x, y, z], initialvalues = [a = 2, b = 1, c = 0], output = [leastsquaresfunction, residuals]\right)$

$$\left[x^{1.14701973996968} - \frac{0.298041864889394\,x^2}{y} - 0.0982511893429762\,y\,z,\right.$$
$$[0.0727069457676300, 0.116974310183398, -0.146607992383251,$$
$$\left.-0.0116127470057686, -0.0770361532848388, 0.0886489085642805]\right]$$

APPENDIX B

Introduction to MapleSim Software

Abstract: MapleSim is a modelling environment for creating and simulating complex multi-domain physical systems. It allows building component diagrams that represent physical systems in the graphical form. MapleSim automatically generates model equations from the component diagrams using symbolic and numerical approaches and runs very highly accurate simulations.

MapleSim modelling environment combines components from different engineering domains such as mechanical, electrical, and multi-body for building and exploring realistic designs and for studying the system level.

Keywords: Maple, MapleSim, Monte Carlo Simulation, Numerical, Symbolic, Simulation.

INTERACTIONS

In MapleSim environment

- Models' system level can be easily assessed to demonstrate concepts such as parameter optimization, sensitivity analyses, and interactions.

- Mathematical equations can be defined for new components from the first principle.
- Simulation can be carried out to investigate a much larger result of conditions that is possible. By testing with hardware alone, with no risk of damage to the equipment and for less cost.
- Allows export from MapleSim to C code, simulation, Labview, and other tools where it can be incorporated with a physical prototype.

Features in MapleSim

- MapleSim have facility for visualization in 3D and animation of multibody systems, full playback, and cameral control in 3D visualization.
- Interface and modelling: It contains drag-and-drop block diagrams in modelling environment, modelling diagrams, and 3-D model construction of multibody systems, data import, and export.
- Block Library: MapleSim contains both physical component and signal-flow blocks. The physical component blocks have different formalities for many domains.

Benjamin Oyediran Oyelami

- Analysis and documentation: extract, view, and manipulation of the system equations for a model l, and parameter optimization. Simulation and parameter swaps including related files in a MapleSim model for easy documentation management and sharing.

Linear, nonlinear, continuous and discrete, SISO, MIMOS and hybrid systems parameter set managing and deployment to popular platforms from Mathword. MapleSim Connect can connect with Simulink.

B1. Design of Simulation using the MapleSim

B1.1 Code Generation

Code generation can handle all systems modeled in MapleSim, including hybrid systems with defined signal input (RealInput) and signal output (RealOutput) ports (MapleSim).

The source code in MapleSim is designed to interface with Maple, in the sample code; all inputs are assigned the value of 0. For more information about the available Code Generation command, see the GetCompiledProc help topic in MapleSim.

C Code Generation

For C code generation, select the attachment of the generation of code from the MapleSim.

Step 1: Subsystem Selection

Click the button:

Step 2: Inputs/Outputs and Parameter Management

Inputs:

	Input Variables	Change Row
1		

Outputs:

Toggle Export Column

	Output Variables	Export	Change Row
1			
2	`Main.'output 1'.T`(t)	"X"	
3	`Main.output2.T`(t)	"X"	
4	`Main.output3.T`(t)	"X"	

☐ Add an additional output port for subsystem state variables

Parameters:

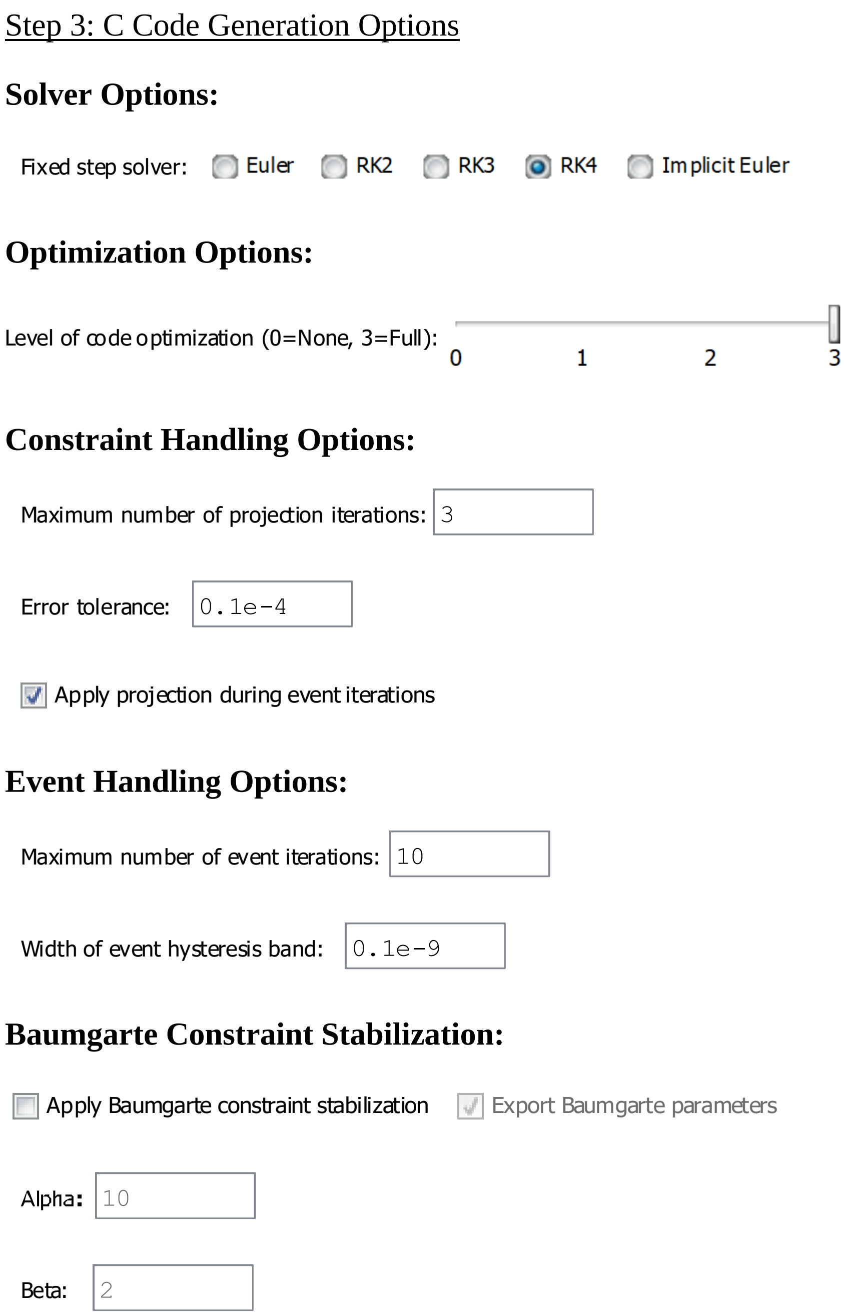
Click:

Toggle Export Column

Then, the parameters used in the model would be generated as:

	Parameters	Value	Export	Change Row
1			"X"	
2	HC1_C	15.	"X"	
3	HC2_C	15.	"X"	
4	HC3_C	15.	"X"	
5	TC1_G	0.1e2	"X"	
6	TC2_G	8.	"X"	

The C code for the modeling program can be generated using various solvers by selecting optimization optional and the max mean projection iteration


Step 3: C Code Generation Options
Solver Options:
Fixed step solver: Euler RK2 RK3 RK4 Implicit Euler
Optimization Options:
Level of code optimization (0=None, 3=Full):
0 1 2 3
Constraint Handling Options:
Maximum number of projection iterations: 3
Error tolerance: 0.1e-4
Apply projection during event iterations
Event Handling Options:
Maximum number of event iterations: 10
Width of event hysteresis band: 0.1e-9
Baumgarte Constraint Stabilization:
Apply Baumgarte constraint stabilization
Export Baumgarte parameters
Alpha: 10
Beta: 2

Step 4: Generate C Code

Target directory:

`C:\Users\Prof B O Oyelami` Browse

C-File:

`MsimModel`

Click to generate the C code:

Generate C code

Step 5: View C Code

```
/****************************************************
 * Automatically generated by Maple.
 * Created On: Fri Jun 12 04:49:35 2015.
****************************************************/
#ifdef WMI_WINNT
#define EXP __declspec(dllexport)
#else
#ifdef X86_64_WINDOWS
#define EXP __declspec(dllexport)
#else
#define EXP
#endif
#endif
#include <stdlib.h>
#include <stdio.h>
#include <math.h>
#ifdef FROM_MAPLE
#include <mplshlib.h>
static MKernelVector kv;
EXP ALGEB M_DECL SetKernelVector(MKernelVector kv_in, ALGEB args) { kv=kv_in; ret…
#else
#ifdef WMI_WINNT
#define M_DECL __stdcall
#else
#define M_DECL
```

Monte Carlo Simulation

Author : Benjamin O Oyelami

Date:14 October 2022

Model Description

Monte Carlo simulation (MCS) can be made on a MapleSim model. To generate MCS, you define a random distribution for a parameter and you can run multiple simulations using this distribution. Note that the properties that are plotted are defined by the probes in the MapleSim model.

Monte-Carlo Simulation

To start, click **Load System**.

Load System

Parameter Distribution

Select the parameter you want to vary, and then choose an appropriate distribution and distribution parameters.

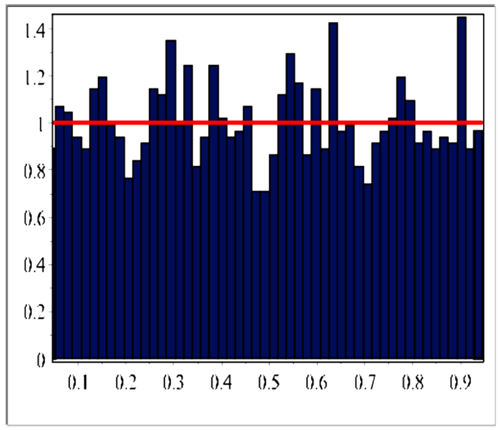
Parameter HC3.C

Nominal Value 15

Distribution Uniform

Choose the parameter a and b, the Help

uniform distribution as follows:



a 0

b 1

Monte-Carlo Simulation

Enter the number of simulation runs and the number of bins in the simulation, and then click the **Run Simulation** button to create and display the simulation plots.

Number of simulations run (including nominal value) 6

To plot variation, in the boxes click:

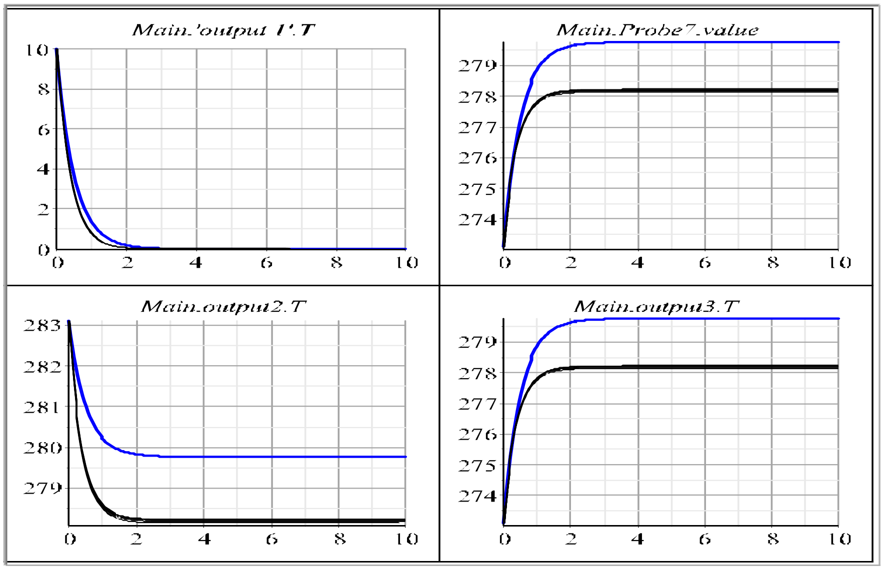
Run Simulation for All probes Plot variances in boxes

Click Run simulation for the given problem and number of bins and the probe plots are displayed above: 12 **Number of bins**

Note: The blue line corresponds to the nominal values.

Data Analysis

Specify a time value below (any float value between 0 and tf s), choose an output variable in the list, and then click **Analyze Data**. Statistics quantities will be displayed on a data set of 5 points, with each point corresponding to one of the simulations, not including the nominal. More information on the quantities displayed and plotted; see Statistics in the Maple Help. The data on which the quantities are computed and plotted are stored as a list of Matrices in the variable _data . The first element corresponds to the nominal value (which is not used in the statistics). Select the desired sample of output variables and click analyze data and the statistics quantities displayed as follows:



Sample Time 5 Analyze Data

Output Variable Main.'output 1'.T

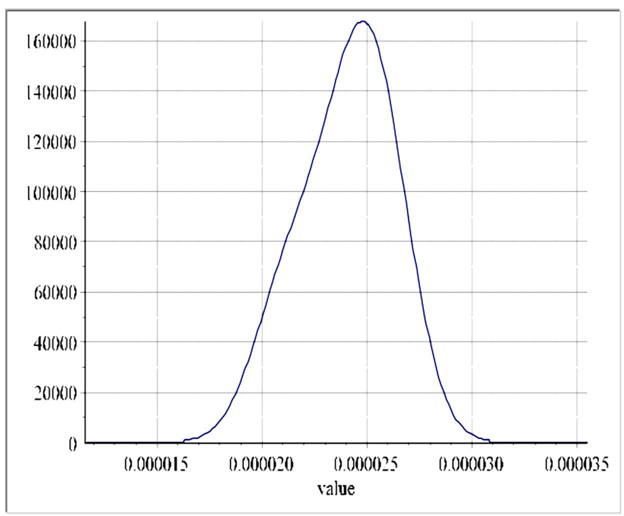
Statistics Quantities

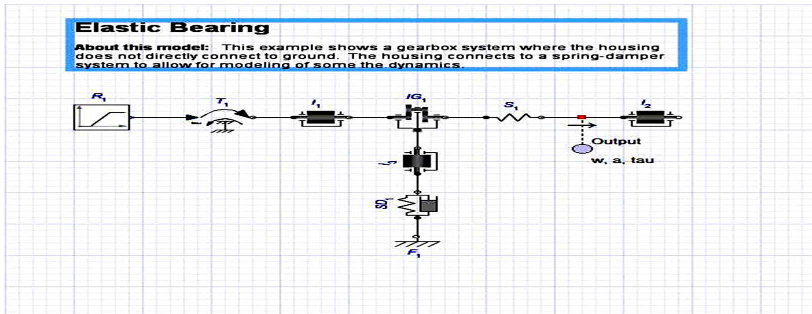
```
0.000005
Skewness
-0.517357
Standard Deviation
0.000002
Variance
0.000000
Variation
0.084149
```

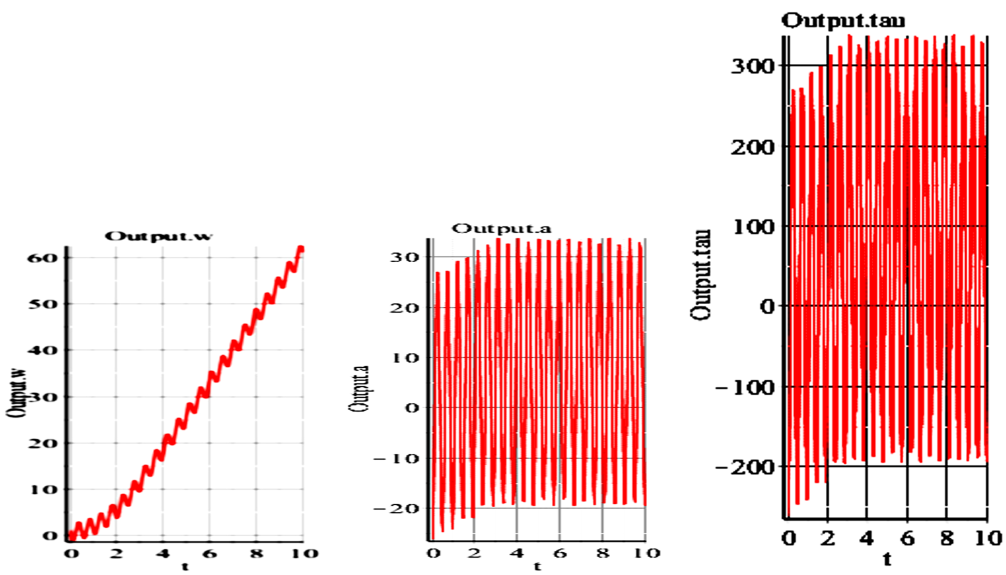
Select the type of plot you desire and click on the example,

Choose the Kernel/density plot and the plot displayed as follows:

KernelDensityPlot







Save this worksheet in Maple and then save the **msim** file to which this worksheet is attached in MapleSim.

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