DESIGN AND CONSTRUCTION OF BUILDINGS AND FOUNDATIONS WITH ILLUSTRATIVE EXAMPLES

Y.M. Cheng
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Design and Construction of Buildings and Foundations with Illustrative Examples

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PREFACE

With the increase in commercial and residential needs, particularly in Asia, more taller buildings are being designed and constructed. Tall buildings are expensive and involve many considerations in both the design and construction processes. The associated foundations for tall buildings may be constructed in different ground conditions, from good to bad soil/rock conditions. To the surprise of the authors, most of the existing books on building and foundation design and construction are superficial to different extents. While some of the basic structural analysis and design principles are generally covered in these books, there are more details on the modelling of structures and foundations, which are not well covered. To be more specific, there are major gaps between what is described in most of the books and the actual design practice required by the engineers. This is the feedback about many existing books written by various graduate engineers.

The authors have extensive practical and research experience in building and foundation engineering and have also been involved in some of the design code/handbooks in Hong Kong. In view of the limitations of most of the books, the authors have tried to cover some of these deficiencies in their teaching materials by providing adequate bridges between theory and actual design practice. The authors have also noted that many design practices, methods, equations, and figures used by the engineers are never clearly explained. Many graduate engineers simply follow these common design practices blindly without adequate understanding. In fact, one of the examiners for the structural discipline of the Hong Kong Institute of Engineers has expressed the concern that many engineers can operate computer software well but lack the knowledge of the analysis. The authors do not view this situation as beneficial, and this is the background for the preparation of this book.

In this book, the authors try to provide adequate information of the basic theory of construction (or references to the basic theory) and, a clear explanation of the design principles and design practice, with some advanced topics and updated theory of building construction. It is hoped that the gap between what is described in this book and the actual engineering practice can be reduced so that an engineer can pick up the design practices easily with a good understanding of structural principles. Since many engineers adopt the use of different computer software (mostly finite element-based), the limitations and precautions in using computer software will also be discussed in this book.

Finally, the authors have taken a real project for illustration of the analysis and design of a multi-storey buildings, together with the basements and foundations for illustration. The complete feasibility report, design analysis and design report, all the required computer input files and drawings are shown. The readers can refer to the reports and drawings for the actual design procedures and construction details for superstructures, basements and foundations. This will greatly facilitate the learning process as well as real design works.

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CHAPTER 1

Introduction

Abstract: This chapter introduces the basic considerations for the structural forms of tall buildings and the history and lists of tall buildings in Hong Kong. A general introduction to the structure of the whole book is also included to facilitate the readers.

Keywords: Buildings, Construction, Code of practice, Design, Standards, Structural forms.

1. INTRODUCTION

With the increase in commercial and residential needs, particularly in Asia, more taller buildings are being designed and constructed. Tall buildings are expensive and involve many considerations in both the design and construction processes. The associated foundations for the tall buildings may be constructed in different ground conditions, ranging from good to bad soil/rock conditions that require careful deliberations. Upon review of a number of existing books on building and foundation design and construction, the authors opine that the basic structural analysis and design principles covered in these books are generally not adequately in-depth. In addition, with the availability of advanced computer methods, analysis based on sophisticated theories and methods that could not be conveniently carried out in the past can now be easily and speedily performed. Examples are large deformation analysis, staged construction, *etc*. These methods are actually required for accurate and realistic analyses for some buildings with difficult structural configurations such as large span transfer structures. The authors consider that this would be the trend of future structural analysis. Nevertheless, to adopt these sophisticated methods, practitioners need to have better knowledge of the underlying theories and principles, a grasp of computer modelling techniques, and an awareness of their limitations. These are, however, not adequately covered in the existing books, as to the authors' knowledge. This book aims at filling these gaps.

2. AIMS AND SCOPE

The authors will furnish fundamental discussions on various design aspects as appropriate in this book with reference to computer methods where applicable.

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Illustrations by practical examples, computer model analyses, and worked numerical examples are added to promote readers' understanding. This book also contains topics not commonly found in other general building design books such as detailed descriptions and applications of wind and seismic loads on buildings, structural floor vibrations, fire resistance design, soil structure interactions, *etc*., and the relevant computer methods that are useful topics for an engineer to build up versatility.

Building structures for residential, commercial, or industrial purposes are the commonest and most numerous structures built by human beings on earth. Compared with other types of structure, a building structure is characterised by its structural form generally in higher complexity and with many structural elements in its superstructure and foundation. These structural elements perform their intended functions and interact with each other which integrate to achieve the structural adequacy, stability, and safety of the building as a whole. A good design of a building structure is marked by its ability to achieve high efficiency in the performance of the individual structural members and that of the building in a holistic manner.

To achieve a good building design, it is, therefore, necessary to have an in-depth understanding of the behaviour of the individual members, their interactions with each other, and their integrated functioning in the building. The understanding has, however, to be based on knowledge of the basic theories and their applications in the analytical methods. Nowadays, with the advanced use of computer methods, good knowledge of computer modelling and its limitations are also required.

The authors have extensive practical and research experience in building and foundation engineering and have also been involved in some of the design code/handbooks in Hong Kong. In view of the limitations of most of the books, the authors have tried to cover some of these deficiencies in their teaching materials by providing adequate bridges between theory and actual design practice. The authors have also noted that many design practices, methods, equations, and figures used by the engineers are never clearly explained. Many graduate engineers simply follow these common design practices blindly without adequate understanding. In fact, one of the examiners for the structural discipline of the Hong Kong Institute of Engineers has expressed the concern that many engineers can operate computer software well but lack the knowledge of the analysis. The authors do not view this situation to be healthy, and this is the background for the preparation of this book.
This book aims to present a systematic approach to enlighten a structural engineer with the conceptual design of buildings ranging from low-rise to super high-rise,

through discussions on the basic theories and their applicability in practical design. Unlike most of the books focusing only on superstructure or foundation, this book covers both for the readers to have a holistic understanding of building design.

3. HIGHLIGHTS

The following elements of the book are highlighted:

(i) Apart from discussions on the general theories and their applications to building analysis and design, reference is also drawn to practices and requirements by popular codes of practice and standards that aim at facilitating a designer in producing a practical design.

(ii) In addition to discussions on the modern design approaches, a review of some important traditional approaches in the pre-computer era has been included which helps readers to apprehend the evolution of the building structural analysis and design methods. As these methods are performed by manual calculations, while understanding these methods, readers have better chances to know and appreciate their underlying principles, in contrast to the practices with the extensive use of computer methods nowadays, which are often in total black boxes for a designer.

(iii) In the discussion of the various building systems, typical worked examples have been included for readers to grasp the design concepts in a quantitative manner.

(iv) The origins of common types of loads applying on a building structure are discussed, together with the determination of the magnitudes of the loads with respect to the underlying theories including dynamic loads on floors creating vibrations and seismic loads determined in accordance with the popular response spectrum method.

(v) In-depth discussions started with well-known theories about buckling and P-Δ effects are presented, leading to an understanding of the basic concept of structural analysis of buildings and its correct application other than the conventional first-order linear analysis.

(vi) Discussions on design for ductility, robustness, durability, and fire resistances are included, which are often not found in books for general building design. The design concepts of these topics are now dispensable in building structural design.

(vii) Fundamental theories and the underlying principles are contained in foundation analysis and design. These theories and principles cover the classical ones by simple calculations adopting rigid cap/rigid footing to the

CHAPTER 2

Structural Design of Building's Superstructure

Abstract: This chapter introduces the various structural elements and systems that are commonly adopted for the structural design of buildings, especially tall buildings. The advantages and limitations of each system are discussed with respect to the structural behaviour and performance. Sufficient in-depth discussions are provided so that the readers can make choice(s) on the structural system(s) that is suitable for a particular project.

Keywords: Beam, Braced tube, Column, Core wall, Frame, Ground conditions, Loadings, Out-rigger, Precast, Prestressing, Reinforced concrete, Slab, Structural systems, Shear wall, Tube.

1. INTRODUCTION

The structural system of a building refers to an assembly of structural members so arranged as to achieve structural integrity, serviceability, and stability for the building. The system can also exhibit certain characteristics for some intended purposes of the building which may be residential, commercial, or industrial. Take an example of a theatre requiring a large space clear of columns resulting in large span structures, truss floor structure may be a feasible structural system.

The main factors that affect the choice of the structural systems for a building, apart from its intended use which may dictate its internal layout, include its height and plan dimensions. External factors comprising ground conditions, wind climate, and regional seismicity are also factors that cannot be ignored. The choice of the most appropriate system for a building structure is very important. As otherwise, an inappropriate structural system may not be able to meet all the constraints and/or conditions the building structure has to satisfy or be satisfied in an effective or economical manner.

The common structural systems and factors affecting the choice of a building are discussed in the following sections.

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2. FACTORS AFFECTING THE CHOICE OF STRUCTURAL SYSTEMS FOR A BUILDING

The following factors are listed and discussed. They are actually interrelated in the final choice of the most appropriate structural system or combination of systems for a building [1].

2.1. Building Use and Architectural Layout

Under a given architectural layout designed for the intended purpose of the building, the column wall layout may be more or less prescribed which would also dictate the span lengths of the floor structures. The structural engineer will make the best use of the available columns and walls and check if they are adequate to carry the imposed loads comprising gravity loads and lateral loads. Conventionally all the columns and walls should be continuous throughout the floors down to the foundation. However, in case of discontinuity to suit architectural layouts, the consideration of "transfer structure" and/or "hanging structure" may be required. The latter is less preferred as the removal of the temporary work has to be deferred to the completion of the supporting structure on the upper floors.

The next step will be the choice of the floor system with span, headroom, and imposed loads as the determining factors. Ordinary beam slab floor systems should be used for normal spans depending on the available headroom and imposed loads which are more economical and with high structural efficiency. Otherwise, a flat slab, or voided slab may be used due to headroom or aesthetic reasons. The roof slab of the upper decked bus station in Mong Kok Mass Transit Railway Station of Hong Kong as shown in Fig. (**2.18**) is a voided slab structure built with limited structural depth compared with the relatively large column spacing and at the same time serves an aesthetic purpose. In the case of large spans, pre-stressed concrete, composite (concrete and structural steel) construction or truss structures may have to be considered.

2.2. Ground Conditions

Ground geology affects primarily the choice of the foundation system. Nevertheless, poor ground conditions by which the building may be prone to excessive settlement and/or differential settlement may affect also the choice of the structural system of the superstructure. As such, the stiffness of the superstructure may have to be "tuned" to minimize the adverse effects due to settlement/ differential settlement. "Hinged" connection of the floor system to the supporting wall/column may have to be designed for some appropriate locations to avoid cracking or failure with the allowance of the comparatively large settlement /differential settlement for the locations. At other locations, stiffness may have to be enhanced instead so as to reduce differential settlement. There are many transfer structures that are designed to be very stiff so as to reduce differential settlements of the superstructures they support, *e.g*. a public housing development in Yuen Long of Hong Kong with thick transfer structures for the purpose as the ground geology comprising marbles and cavities is very poor.

2.3. Loads

Loads on buildings generally comprise gravity load, wind load, and seismic load. The former is applicable in all buildings while the latter two depend on the locality of the building. In reinforced concrete buildings, the building's own weight contributes to the predominant part of the gravity loads while the live load has generally a much lesser contribution. Heavy load sometimes affects the choice of a structural system, say requiring pre-stressed concrete instead of reinforced concrete frames to resist the gravity loads. For example, the commercial development over the West Kowloon Terminus of the Express Railway Link in Hong Kong with over 12m span of flat slab construction carrying relatively high loads adopts a pre-stressed concrete slab of 325mm thick to reduce deflection and bending.

Wind loads and seismic loads are mostly lateral loads creating not only internal forces within the building structures but also lateral displacement and vibration which are also design parameters for the building. These loads are dynamic in nature and their magnitudes for design on the building depend also on the dynamic properties of the building which govern the building's response to them. For ease of design, these loads are mostly converted into lateral static (pseudo-static) loads acting on the building, though dynamic analyses (time history analyses) are sometimes required for very "slender" buildings for accurate and economical design. Thus high wind loads generally require the building to possess adequate lateral stiffness to keep lateral deflection within pre-determined allowable limits. This criterion often governs the choice of the structural system by which frames comprising relatively stiff walls under good "coupling" are required instead of the column beam frames with much lesser stiffness. The magnitude of the direct wind pressure on a building depends primarily on the locality of the building, *i.e*., whether it is a high wind zone (tropical or sub-tropical). In addition to the magnitude of the direct wind pressure creating primarily the static response, the dynamic response of the building to wind loads depends on the closeness of its fundamental natural frequency to that of the natural wind which can be broken down into a

CHAPTER 3

Loads on Superstructure and their Statistical Nature

Abstract: This chapter discusses the various types of loadings that are required to be considered in the design of buildings. The loadings include dead loads and various types of live loads, wind loads, water and soil loads, seismic loads, snow loads, temperature loads, and others. There are various considerations behind these loadings, which are discussed in depth in this chapter. Various load combinations will also be discussed with respect to British codes, Euro codes, and ASCE codes.

Keywords: Dead load, Dynamic load, Live load, Load combinations, Load pattern, Response spectrum, Seismic load, Sheltering effect, Shrinkage and creep, Topographical factor, Wind load, Wind tunnel.

1. INTRODUCTION

For the intended purpose(s) of a building, which may be residential, commercial or industrial, inevitably, the building has to be designed to carry the loads due to the intended purpose(s). These loads that are mostly gravitational in nature are generated from human occupants, furniture, machinery, equipment, *etc*. They are termed live loads as they are movable and the magnitudes may vary. The building, by its construction materials, also constitutes another gravitational load known as the “dead load” because they are relatively constant and stationery throughout the life of the building.

In addition to the dead and live load that a building has to be designed to withstand, other load types that have to be considered for design include wind load, seismic load, soil load snow load, *etc*. Out of these loads, dead load as the “permanent action” related mostly to the weights of the structures, can be determined generally with high certainty as the densities of the construction materials can be found out precisely. However, other loads are less certain which generally have to be determined through statistical approaches.

It would be ideal if the loads on a structure such as wind load, seismic load, and snow load, can be predicted through the laws of nature known by scientists. However, such predictions to precise levels are not possible at the present knowledge level. So, these actions are currently treated as “random actions”. The magnitudes and occurrence frequencies of them have to be estimated by statistical

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analyses of past data. The estimated results for application in engineering design are often in the form of magnitudes associated with probabilities of occurrence. On the other hand, though the live load is similarly treated, it can, however, be determined much more precisely as the loads are artificially imposed instead of naturally occurring. The variations of live load are comparatively small. Generally speaking, more sophisticated statistical approaches are often employed for the determination of wind and seismic loads as they are still highly unpredictable. These methods would give higher design values above average than that of live load. For example, "extreme value analysis" with longer "periods of return" is often employed for wind and seismic load while that of live load may be taken as the characteristic values with not more than 5% opportunity of exceedance under the assumption that the statistical data are in normal distributions.

Soil loads are basically determined by the knowledge of engineering of the behaviour of soil. However, natural occurrences such as effects due to seismicity and rainfall that may affect slope stability and the like may also involve statistical contents. The uncertainties are generally covered by the use of conservative parameters or higher factors of safety.

The various types of loads will be described in turn with regard to their nature, quantification, and application on the structures. Finally, a combination of them for practical structural design is discussed.

2. DEAD AND LIVE LOAD (IN GENERAL)

Dead load and live load are gravity loads acting in the vertical direction. Dead loads (permanent actions) of a structure are those that are constant or stationery throughout the life of the structure. They normally refer to the weight of the structure and the permanent attachments such as finishes, partitions, services, *etc*. The determination of dead load can be of high certainty as it is based on the densities of the materials and pre-determined layouts which can generally be found out precisely. However, as the precise layouts of these permanent attachments may not be known in the design stage or there can still be variations after building occupation, conservative values are generally adopted in design to cover the uncertainties. Some codes of practice recommend design values for them. For example, the Code of Practice for Dead and Imposed Load 2011 [1] of Hong Kong, making reference to the LCC By Law 1968 [2], recommends a partition load of 1/3 of the weight per meter length of the partition as uniformly distributed load per square meter or not less than 1 kPa for office if the partitions are not indicated on the building plans.

Live loads (variable actions) acting on a structure refer to those that are not constant and change with time. They normally include weights of occupants, furniture, machinery, and the like. Although these weights are not constant, they however vary within some ranges which can be determined by statistical approaches. Design codes stipulate values for design purposes with reference to the intended purposes of compartments of building structures. For example, $2kN/m^2$ or $3kN/m^2$ is stipulated for the residential and office areas of residential and office buildings which include occupants, furniture and equipment; 3 to $4kN/m^2$ for carparks due to the weight of the cars, *etc*. These design values are often taken as the characteristic values with a 5% probability of being exceeded under the assumption of normal distribution. That is, it is the mean + 1.64 times the standard deviation. This approach is adopted by the Eurocode BSEN 1990 [3], the Australian Load Code ASNZS 1170-1 [4], and the Hong Kong Load Code 2011 [1]. The American load code ASCE7-16 [5]; however, requires the load to be the "maximum expected values". The characteristic load concept can also be applied to dead load, where the standard deviations are generally small so that they are close to the mean values. Guidance can also be found in BD-006 [6].

The dead loads are, without a doubt, static loads to be applied to the building structure. Live loads such as those due to the moving occupants and machinery under working conditions may, strictly speaking, be dynamic. However, they are mostly treated as static loads in the analysis of building structures, with some factors to account for the dynamic effects as appropriate. It will be seen that loads that are truly dynamic, such as the wind and seismic loads, are also often treated as static loads in building structure design. Nevertheless, in case the dynamic nature of the load is very pronounced, dynamic analysis of the structure may be required, which is mainly the effects due to vibration.

The calculation of dead and live loads on a structure are generally through the direct application of surface loads, line loads or point loads, simulating the loads as appropriate onto the structural members. For example, the live load or finishing load on a slab may be taken as a uniformly distributed surface load on the slab as live load and dead load, respectively. A partition resting on a length of a beam may be taken as a uniformly distributed line load on the beam within the length as dead load. Load from a secondary beam on the main beam maybe taken as a point load. A practical example of the application of dead and live load on a continuous beam is given in Appendix C1.

CHAPTER 4

Approaches and Methods for Analysis of Building Structure

Abstract: This chapter introduces the classical methods as well as the modern computational approach for tall building analysis. The structural behaviour of basic structural elements as well as sub-systems and joints are discussed in detail. The second-order effect with respect to large deformation which is important for steel structures is also discussed in this chapter. The materials in this chapter can greatly enhance the application of computer programs for the analysis of complicated tall buildings.

Keywords: Buckling, Construction sequence, Elastic analysis, Load path, Lateral load analysis, Moment distribution, Rigid and semi-rigid joint, Rigid and semi-rigid diaphragm, Second order effect.

1. INTRODUCTION

A building structure is a relatively complex structure comprising many structural members interacting with each other. So compared with other types of structures, very often greater extents of simplification and idealization in structural analysis have to be employed in order to achieve a realistic design. The simplifications often involve "breaking the whole structure" into parts which may be termed "sub-frames" and the idealizations generally include simulations of the 3-dimensional structural members by line or plate members, idealization of support conditions of being "pinned" or "fixed" for the whole structure and/or the individual members, *etc*. Other assumptions including "elastic analysis", "rigid joint connection", and "small structure deflection" are also commonly employed. As such, some shortcomings including incompatibility of deformations/stresses (due to sub-frame analysis), extremities in support, and/or member end forces (due to pinned, fixed, or rigid joint assumption) often have to be tolerated. Nowadays, these shortcomings can be minimized under the use of computer methods, though such minimizations are still not very popularly employed.

In this chapter, the common assumptions in analysis and design are first discussed, followed by a historical review of the structural analysis and design in the pre-computer era. More in-depth discussions are then presented for the various modern analytical approaches.

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2. COMMON ASSUMPTIONS IN STRUCTURAL ANALYSIS AND DESIGN FOR BUILDINGS

2.1. Elastic Analysis – First-Order Linear with Small Deformations

Under "elastic analysis" (or first-order linear analysis) of structure, deformation of the structure is assumed to be linearly proportional to the applied load with no deformation limit. The proportionality is taken as the stiffness of the structure as:

$$K_e.\Delta = F \qquad \textbf{(4.1)}$$

Where Δ is the deflection, F is the applied force and K_e is the elastic stiffness, which are generally in matrix form as they generally comprise large number of degrees of freedom. The simplest example is a cantilever beam idealized as a line member of length L lying in the X-direction with 3 degrees of freedom at its free end:

$$\begin{bmatrix} AE/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} \qquad \textbf{(4.2)}$$

Where E is Young's modulus of the material; A is the cross-sectional area and I is the second moment of area of the beam section; δ_x, and δ_y are the linear deflections in the X and Y direction, respectively ; θ_z is the rotation about the Z direction; F_x and F_y are the applied forces in the X, Y direction, respectively; M_z is the moment about the Z direction.

The assumption is generally valid when the internal stresses in the structural members are well below the elastic limits of the structural materials (concrete and steel *etc*.) which generally behave elastically. Under such an assumption that the deformations are small, the "principle of load superposition" can be employed with adequate accuracy. The principle also implies that the load effects including deformations and internal forces of the structure induced by two or more loadings in a linearly elastic structure are equal to the sum of the load effects caused by the individual loadings. It can be implicitly proved by (4.1) where K_e is a constant for a structure as:

$$K_e.\Delta_1 = F_1 \qquad \textbf{(4.3)}$$

$$K_e.\Delta_2 = F_2 \qquad \textbf{(4.4)}$$

Where Δ_1 is the deformation due to F_1 and Δ_2 is the deformation due to F_2. Adding (4.3) and (4.4) together

$$K_e.(\Delta_1 + \Delta_2) = F_1 + F_2 \tag{4.5}$$

which proves the load superposition principle.

Another implicit assumption taken in the analysis is that the applied loads are assumed to remain unchanged in position when the structure deforms, or else K_e will not remain constant due to the change of the geometry by the load. Consider a cantilever beam under an axial load and lateral load as shown in Fig. (**4.1**). The analytical result for moment at the fixed end by the first order linear analysis is only $M = SL$, irrespective of the lateral deflection Δ at the free end which obviously will create an additional moment of $P \times \Delta$ famously known as the P-Δ effect.

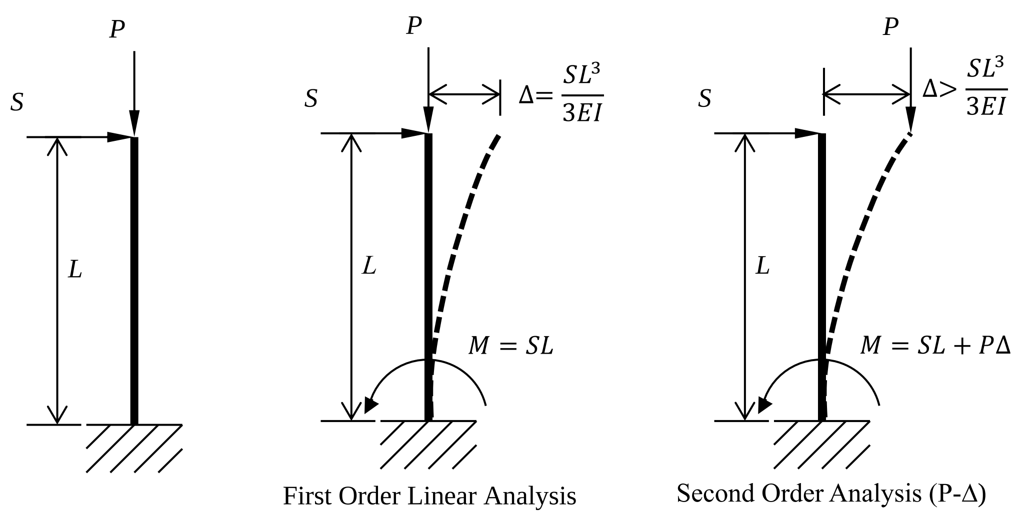

Fig. (4.1). Illustration of P-Δ Effect.

To vigorously cater to the P-Δ effect, K_e will have to be changed as the geometry of the structure changes. But this is generally ignored in the elastic (first-order linear) analysis. This is based on the assumption that the deformations of the structure are generally small. However, if the structure is flexible and/or the applied loads are of high magnitudes such that the deflections are large, large eccentric moments may be created which would result in significant errors as compared with that of the first-order linear analysis. Despite the potential errors, the first-order

CHAPTER 5

Ductility, Robustness, Durability and Fire Resistance Design Considerations in Buildings

Abstract: This chapter discusses the ductility, durability, robustness, and fire resistance considerations for building designs which are usually not well-treated in many texts. The considerations will include both reinforced concrete and steel buildings.

Keywords: Concrete structures, Ductility, Durability, Fire resistance, Robustness, Steel structures.

1. INTRODUCTION

Other than strengths and deformations due to imposed loads, which are the prime concern in building structure design, there are some other important considerations including ductility, robustness, durability and fire resistance. These are discussed in this chapter as related to concrete and steel structures.

2. DUCTILITY

Ductility is defined as the ability of the structure to tolerate large deformations before failure. Such an ability can provide warning to the user of a structure to repair or to escape by the noticeably large deformation; it is a desirable property and its importance has been highly ranked in recent decades. Fig. (**5.1**) shows the difference between ductile and non-ductile (or brittle) structural behaviour and the quantification of ductility. The figure shows a plot of the moments of resistance of two beams, Beam 1 and Beam 2 against their curvatures demonstrating different degrees of ductile behaviours.

The "factor of ductility" can be defined as $\mu = \phi_u/\phi_y$, where ϕ_u is the ultimate curvature taken as that corresponding to 80% of the maximum moment of resistance in the post maximum region and ϕ_y is the yield curvature corresponding to 75% of the maximum moment of resistance in the pre-maximum region as demonstrated in Fig. (**5.1**). It can also be seen that by the slopes of moment of resistance *versus* curvature for both beams in the post maximum regions, Beam 2 is much gentler than Beam 1. The difference is also quantified by their ductility factors μ with $\mu_2 > \mu_1$. So Beam 2 can tolerate a larger curvature (or angular rotation) as deformation than Beam 1 before failure, even though Beam 1 has a higher moment of resistance.

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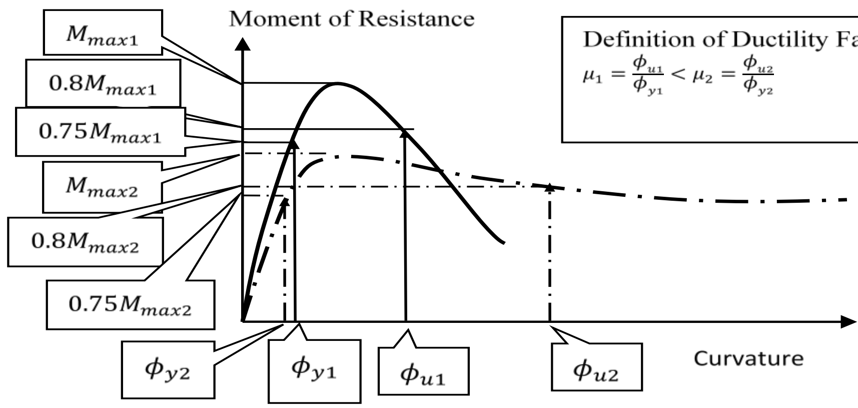

Fig. (5.1). Ductility of beams (moments of resistance varying with curvature).

Considering ductility as an important design parameter, it is thus desirable for a structure to attain certain levels of ductility so that its failure will not be all of a sudden, which can then give warning to the users to repair or to escape. The requirement for ductility is more important in seismic structural design where extensive structural damage or even collapse can occur. The levels of ductility required also depend on the severity of the seismic actions (which vary from location to location) and the importance of the structure.

2.1. General Ductility Requirements

In addition to the comparatively large tolerable deformation required prior to failure, ductility design may also be associated with preferred modes of failure, especially under seismic actions. A preferred mode of failure refers to the preferred sequence of failure of structural members, *e.g.*, beam failure takes precedence over column. The ductility requirements are typically listed as:

(i) The ductility requirement of a structure is quite related to the structure's ability to withstand vibration (mostly during earthquakes) which can effectively be catered by dissipating the vibration energy through the movement of the structure. Such dissipation of energy can be most effectively achieved through "plastic deformation" at the joints under a process of "hysteresis" as demonstrated in the force *versus* displacement diagram in Fig. (**5.2**). The shaded area represents the energy dissipated in a "cycle" of movement. As the movement can be in many cycles, energy will be dissipated several times until the movement stops. So it is desirable to have joint design that can undergo such hysteresis as a ductility provision.

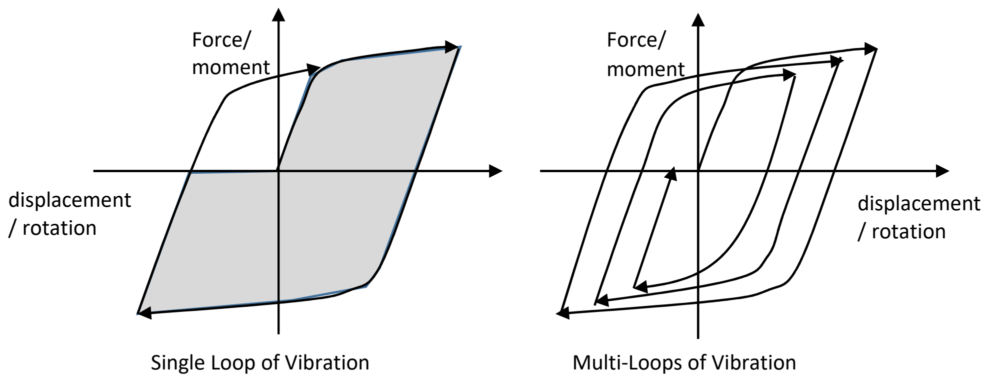

Fig. (5.2). The Phenomenon of "Hysteresis".

(ii) In a building structure, beam failure should take place prior to failure of the supporting column which is the well-known "strong column-weak beam" design concept. This failure mode is preferred as otherwise column failure which takes precedence to that of beam which will be more drastic and can lead to the large area of collapse. Some design codes such as the New Zealand Code NSZ3101 [1] and the China Seismic Code GB50011 [2] require the total moments of resistance of the columns to exceed the total moments of resistance of beams meeting at a column-beam junction by 1.2 times as

$$\sum M_c > 1.2 \sum M_b \qquad \textbf{(5.1)}$$

where M_c and M_b are the moments of resistance of the columns and beams, respectively. This is a "capacity design" concept by which the design of an element should take into account the design strength (capacity) of the adjoining elements instead of only the load it has to carry.

(iii) At a beam-column joint, the structural design of the joint which often fails by shear should be so designed that it is stronger than the connecting beams to avoid joint failure taking precedence to the beams. Such a design approach is also based on the capacity design concept by which the unbalancing moment on the high side shall be based on a factor greater than 1 (1.25 in the New Zealand Code NZS3101 [1]) and on the yield strength of flexural reinforcements.

(iv) "Soft storey" in a building which is a storey significantly more flexible and weak in resisting lateral loads should be avoided. This often occurs at a storey with flexible columns tied to a stiff horizontal floor structure such that the

CHAPTER 6

Criteria for Foundation Options Selection

Abstract: This chapter gives an introduction to the various considerations in the selection of foundations, which include cost, geology, settlement, loadings, and the surrounding buildings.

Keywords: Adjacent structures, Cost, Geology, Loads, Settlement.

1. INTRODUCTION

Similar to the superstructure which can be of different structural forms, there are also different options for the foundations to support the superstructure. The foundation options generally comprise footing and pile as discussed by Bowles [1] and Cheng [2]. Broadly speaking, the selection criteria for the foundation of a building depend on the following factors:

(1) Load intensity from the structure;

(2) Demand for the limitation of settlement and/or differential settlement from the superstructure;

(3) Underground geology;

(4) Limited effects on adjacent structures/installations and features;

(5) Availability of technical know-how;

(6) Economy in construction.

In case conflicts arise in concurrent fulfillment of these criteria, *e.g*. high construction costs will be incurred to limit adverse effects to adjacent structures/installations and features, balance has to be sought by considering all relevant factors with weighting as appropriate for the optimal solution.

These factors are briefly discussed in the following sections with their inter-relations.

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2. LOAD INTENSITY FROM THE SUPERSTRUCTURE

Unless a strong stratum that can offer high bearing capacity exists in a shallow depth, a heavy building generally requires a deep foundation founded at depths where a strong bearing stratum can be met or the summed frictional resistance along the depth of the foundation can safely support the structure. Conversely, low-rise buildings with lesser load intensity can be founded on shallow foundations even if the load capacity of the supporting stratum is not high.

Not only the vertical load intensity is critical for the choice of deep or shallow foundation, but also the lateral load and the overturning moment may require due consideration for high-rise buildings. High over-turning due to lateral load or upthrust may lead to tension on parts of the foundation. In case tension resistance has to be provided to the building, a pile foundation is generally required to anchor the building into the ground. They are generally in the form of pile foundations or strong anchors into the shallow rock if in existence.

In addition to seismic loads, either pseudo-static or dynamic loads obtained from analysis that are required to account for the design of a foundation, some special considerations are also required as appropriate. First of all, a site that is prone to liquefaction in an earthquake should be avoided for building. In seismicity active sites, a close study of the properties of soil should also be conducted, particularly on the changes of properties due to earthquakes such as an increase in pore water pressure leading to the reduction of effective stress, shear strength and stiffness.

Detailed foundation design often takes precedence over the superstructure. So the estimation of loads from the superstructure is often required as the detailed design of the superstructure has not yet commenced. This early-stage load estimation includes (i) gravity load which can be determined by the “tributary area” method; (ii) wind load in accordance with the wind climate of the locality, the building’s shape, and its estimated dynamic properties, *etc*.; (iii) seismic load in accordance with the seismicity of the locality and estimated dynamic properties of the building; (iv) other types of load which may provide control as appropriate. The estimation has to be conventional on one hand but the economy of construction, particularly the foundation option has to be duly considered. The achievement of a balance relies on the experience and technical know-how of the designer. However, innovative ideas should be promoted as appropriate.

3. DEMAND FOR LIMITATION OF SETTLEMENT AND/OR DIFFERENTIAL SETTLEMENT FROM THE SUPERSTRUCTURE

If a superstructure can tolerate little settlement and/or differential settlement, a rigid foundation is required. Generally, a large and thick raft footing or pile cap is stiffer than isolated smaller footings and pile caps. Nevertheless, the rigidity of a foundation depends not only on the rigidity of the structure of the foundation but also on the underground geology. In general, the deeper the rigid rock stratum, the greater settlement will be created to the foundation through settlement of the soil and/or the larger extent of shortening of the long piles which are either founded on soil or the hard stratum. In addition, large variations in the ground geology may also lead to greater differential settlement of the foundation and subsequently to the superstructure which is generally undesirable. The differential settlement can, however, be alleviated under careful planning of the foundation layout with respect to the distribution of the loads from the superstructure. In turn, the superstructure may have to be checked against the effects of the settlement of the foundation and be strengthened in identified locations accordingly.

4. UNDERGROUND GEOLOGY

The most important design parameter of the ground is the “bearing capacity” it can provide to the foundation. For conservative design to cover variations of the ground which may be of high uncertainties, the bearing capacity is often factored by 2 or 3 for support of the foundation. Very often, the bearing capacity is determined by settlement induced instead of punching failure.

Generally speaking, rock can provide a much higher bearing capacity than soil. The bearing capacity can be in the form of direct contact of the foundation with the rock (end-bearing) or frictional/bonding resistance of the foundation with the rock through grouting. However, the bearing capacity of rock depends very much on its degree of weathering and the cracks within the rock mass. There are empirical relations for discounting the bearing capacity of rock with respect to the extent of cracks or fissures. Shallow foundations can often be used in case rock is at shallow depths.

Soil, sandy or cohesive, generally offers a much smaller bearing capacity than rock. However, similar to rock, soil can also provide bearing to the foundation in the form of end bearing, side friction, or a combination of both to the foundation units. Generally speaking, the bearing capacity of a soil depends on its “compactness" which is greater at greater depth. The soil compactness may also be artificially

CHAPTER 7

Shallow Foundation Analysis and Design

Abstract: This chapter discusses the ultimate limit state, and serviceability limit state design of shallow foundation, which is further complimented by site tests. Classical and computational methods for simple footing and raft foundation analysis and design are discussed in detail in sthis chapter.

Keywords: Bearing capacity, Consolidation, Creep, Continuum model, Computer modelling, Displacement, Flexible analysis, Rigid analysis, Stress, Settlement, Wood-Armer design.

1. INTRODUCTION

For the foundation design of a building, there are many considerations which include:

1. The vertical and horizontal loads from the superstructure. The horizontal loads may come from soil, water, earthquakes, winds, loadings induced from adjacent works, and other possible factors. The vertical load includes typically the dead and live load, dynamic loads, earthquake load, and others. In general, the loads will be spread to the ground by the use of footings, mat foundations, pile (or pile raft) foundations, or barrettes. Combinations of these foundation types are also common, as there may be significant differences in the loadings between parts of the buildings.
2. Many tall buildings have basements for different usages, including car parks, shopping areas, and storage areas, and this is very common in many developed cities. A deep foundation or excavation in poor soil in a city area can create many technical difficulties and be expensive in construction.
3. The cost and time of construction of a deep foundation and excavation can be a very critical issue in the construction project. Many construction forms, techniques, and equipment have been developed over the years, and the choice of the construction scheme can be an interesting and important consideration for the project.
4. The choice of foundation depends on the loading types and magnitude, locations, ground conditions, access, time for construction, and other factors. Also, there is usually no unique solution to a problem. For example, the use of

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several large-diameter bore piles instead of many smaller bore piles or driven piles is favoured by some but not all engineers.

5. To reduce the effect of earthquakes on the superstructure, the use of artificial damping is common. The connection between the superstructure columns and the foundations need great care in the design and construction.

As a rule of thumb, a shallow foundation can be adopted if the building is less than 8 floors in height and good soil or rock is located at a relatively shallow depth. Otherwise, a deep foundation will be necessary, except when the rock head is high with the adoption of a raft foundation. This chapter is devoted to the analysis and design of shallow foundations, and deep foundations will be covered in chapter 8.

2. TYPES OF SHALLOW FOUNDATIONS

The depth of a typical shallow foundation is usually within 2 to 3 meters below the ground. The code of Hong Kong [1] defines the depth of a shallow foundation to be less than 3m, while Bowles [2] defines a shallow foundation with a depth to breadth ratio less than one (but may be with somewhat more).

A shallow foundation has a large base area to distribute and spread the vertical loadings and limit the settlement, angular distortion and others under service conditions. A shallow foundation spreads the loadings vertically and horizontally to wider regions while the loading on a pile foundation is taken up mainly by the skin friction and end bearing in the vertical direction without horizontal spread. Actually, settlement is well known to be the most controlling factor in shallow foundation design for normal cases. If the vertical load is close to the ultimate bearing capacity, the settlement under this condition must be large. The control of the applied bearing load can directly control the corresponding settlement, or *vice versa*, and the bearing capacity with an appropriate factor of safety will be deemed to satisfy the settlement requirement in the older design method, when computer and programs are not available. Practically, there occur very few bearing capacity failures for a shallow foundation but many settlement problems.

Shallow foundations are mostly reinforced concrete structures in soil or rock in some cases. The maximum concrete grade is typically limited to Grade 45 for the concrete used in Hong Kong. Most of the buildings in Hong Kong are tall with heavy loads, reinforcement bar diameters ranging from 32mm, 40mm, to even 50mm are commonly used in many constructions projects, but this is less commonly seen in other places. Many engineers also like to use a large foundation instead of many smaller foundations in the design to reduce the labor cost.

The different forms of the shallow foundation include:

(1) A pad footing where the length and breadth of the foundation are comparable. Pad footing usually supports a single column, so punching shear can be a critical issue in the design. Some engineers add a pedestal to increase the depth of the footing locally for punching shear design. Pad footing is used for small point loads with relatively good ground condition.

(2) For a strip footing that supports walls or columns arranged along a line, the length-to-breadth ratio is large.

(3) Two or more footings are sometimes combined together to form a larger footing, or individual footings are indirectly combined by connections with tie beams. Combined footing is usually rectangular in shape, but a trapezoidal shape is adopted if the differences in the column loads are small among different columns. If the columns do not align along a line, it will be termed a raft foundation instead of a combined footing. The advantages of a combined footing are: (1) due to the continuity of the footing, there will be load sharing among the columns so that a sudden major increase in loading on a column can be relieved; (2) raft foundation supporting a series of columns can reduce differential settlement as compared with the use of individual footings.

(4) A raft footing has a large plan area with any possible shape which spans in both directions under the superstructure loadings. Local thickening of raft footing is common for local heavy point or line loads. A raft foundation supports columns and walls as a two-dimensional structure. Currently, most engineers adopt computational methods in the analysis, and plate analysis is the most commonly adopted numerical method at present.

(5) A buoyancy foundation is a buried hollow box structure that aims at minimizing the additional stresses by placing the foundation below ground. The weight of the displaced soil should be approximately equal to the applied load for maximum efficiency such that the net bearing stresses will be small. The settlement in this case will be small and acceptable in general.

The common types of shallow foundations are given in Fig. (**7.1**).

2.1 Different Failure Modes of Shallow Foundation on Soil

Many laboratory tests on bearing capacity and failure mechanisms of shallow foundations under different soil conditions and depths have been performed [2]. The bearing capacity and failure mechanism of shallow foundations are also investigated by many researchers using different types of methods. The model test results, the theoretical studies from the basic governing equation as well as

CHAPTER 8

Pile Foundation Analysis, Design and Construction

Abstract: In this chapter, different types of piles are classified, with the method of construction and design. The analysis and design of piles include vertical and lateral load analysis for single piles and pile groups. A detailed discussion of the various field pile tests and the methods for interpretation are included to assist the readers in the field control of pile installation.

Keywords: Construction method, Computational method, Lateral load, Pile test, Pile group, Pile type and classification, Rigid cap analysis, Single pile, Vertical load.

1. INTRODUCTION

Piles take up vertical loadings, lateral loads and tensions from superstructures. Since piles are installed mainly below ground (a short distance above ground in some cases), many types of piles have been developed for different ground conditions and environmental conditions with the considerations of loadings, cost and time, construction methods, local practices, effects on surroundings and other factors. In many developed countries with multi-storey buildings, pile foundation will be the most common foundation, as it can take up a much larger load as compared with the shallow foundation.

2. CLASSIFICATION OF PILES

There are different ways to classify a pile, which include the types of pile material, load transfer or method of installation and others. The basic classifications for different types of piles:

(a) End bearing *vs* friction pile – This loading classification is based on the load transfer or the distribution of pile capacity. If the total pile capacity is mainly due to the shaft skin friction, it is termed a “friction pile”. If the pile capacity is mainly contributed by the end-bearing capacity at the pile base, it is termed an “end-bearing pile”. This classification is somewhat not precise in that the ratio between the end-bearing and shaft capacity will vary with time and loading. Many end-bearing piles function as friction piles under normal load conditions with only limited loading transferred to the pile base, which has been confirmed by a huge amount of pile load tests in different places. Actually, there is no rigid guideline among the engineers under this classification. Currently, if the design

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shaft friction of a pile exceeds the pile end bearing capacity, that pile will be termed as a friction pile, irrespective of the load distribution under working conditions.

(b) Percussive and non-percussive pile – which is based on the method of pile construction. A pile can be installed by percussive driving (driven pile) or by forming a hole (bore pile) by augering or rotary cutting action. A pre-formed precast concrete pile or steel pile is sometimes placed inside the hole, or more commonly reinforcement cage with concrete or grout poured into the hole to form the pile.

(c) Displacement pile (large to small displacement) or non-displacement pile – This classification is based on the volume of displaced soil during installation. Large displacement pile creates relatively large soil displacement and vibration/noise hence it is less popular nowadays. Small displacement pile generates smaller displacement/vibration of soil during the installation, and steel H or hollow section piles are the representative small displacement piles. Non-displacement or non-percussive piles are formed by boring before the pile construction. The amount of soil displacement and the use of pressure during concreting (if any) will determine the working and ultimate skin friction along the shaft and the possible loosening/ densification of the surrounding soil.

(d) By materials:

Timber pile – seldom used nowadays, but many existing old structures are supported on timber piles.

Concrete pile – usually in the form of precast piles (reinforced and more commonly prestressed) or cast-in-situ piles.

Steel pile – mostly tubular or H pile, but piles with other shapes are sometimes used.

Composite pile – a combination of two or more structural materials, and most commonly tubular steel pile with concrete or cast in-situ concrete pile with steel section. This form has been popular in some mass transit concourse construction in Hong Kong.

Grouted pile – the pile is formed by the use of grout (mostly cement grout) with pressure. The pressure may be nominal or high pressure, and the quality of the pile depends heavily on the grouting pressure.

(e) Loading mode - Axial compression or tension load, transverse or lateral load, moment or combinations of these loadings may be taken up by the piles. The loadings from the superstructures will be taken up by the pile cap directly and transferred to the pile.

(f) Shape – H section, solid/hollow circular or square section, octagonal (solid or hollow) and others.

(g) Miscellaneous – piles may have constant cross-section or tapered, and on some occasions, may have different sizes. For some projects that require the use of very long piles, some engineers will use heavier pile sections for the top while reduced sections for the rest of the pile to control both the settlement and cost of construction.

The final choice of the actual deep foundation depends on many factors, which include time, cost, construction plants, effects on surrounding and other factors. There is an interesting case in Hong Kong to illustrate the importance of construction plants and the cost of the final design. For the Central Plaza construction in Hong Kong, diaphragm walls and bore piles are specified in the drawing. To avoid mobilization of different plants, the contractor proposed to replace the bore pile with barrette, which has achieved a major saving in the construction cost. The Odex drill in Fig. (**8.3a**) has caused several major ground loss and settlement cases in Hong Kong and Macau. Hence, it is now replaced by the use of a concentric drilling system in Fig. (**8.3b**). The use of driven H piles is replaced by the socket H-pile in sites with many boulders; the use of bore piles instead of driven piles for projects with adjacent old buildings; the use of mini piles instead of large diameter bore pile for sloping areas, and other cases are good illustrations for the proper choice of deep foundation with considerations of ground conditions and surrounding buildings. A good, deep foundation has to consider all the factors mentioned above, and the engineers should have a good understanding of the applicability and limitations of different piling systems before making the final decision on the piling system. Section 8.2 provides some discussions about some commonly used piling systems, which can help the readers select a suitable system in normal cases. The experience of the piling contractors, past case history and failure cases should also be considered in the design process. Table **8.1** is a table that illustrates the advantages and limitations of some pile types, while the details of each pile system are discussed in section 8.2.

CHAPTER 9

Excavation and Lateral Support System (ELS)

Abstract: In this chapter, different types of retaining wall systems are introduced. The method of installation of these wall types is discussed in detail, with the support of many field photos and construction method statements from contractors. The methods of excavation are illustrated with some practical examples from Hong Kong. Following the construction aspects, the theory behind the lateral earth pressure, water pressure and the methods of analysis for the staged excavation are discussed, while the problems that may arise during deep excavation are illustrated with practical examples. A sample monitoring scheme for a project is also included in this chapter to help the readers appreciate the use of IOT monitoring during construction, which is now becoming popular for many large-scale construction works.

Keywords: Analysis, Bottom-up, Finite element method, Installation, Instability, Lateral earth pressure, Monitoringm, Retaining wall types, Settlement, Subgrade reaction, Top-down, Water pressure.

1. INTRODUCTION

In many developed cities, deep excavations are associated with many tall buildings, subways, mass transit and other construction works. ELS work is actually one of the major construction works in Hong Kong. Due to the complicated ground and environmental considerations, many support systems have been developed in Hong Kong. In Hong Kong, ELS plan submission is required when the excavation is longer than 5m and deeper than 4.5m. In general, the ELS submission includes:

1. Plan and sections for the ELS, *e.g.*, pipe piles, sheet piles, diaphragm wall *etc*.
2. Support layout in the form of struts or tie-backs, sections and connections and other necessary details.
3. Construction sequences of the ELS and special arrangement.
4. Supporting documents, including the site investigation reports and study about the surrounding buildings, utilities *etc*.
5. Analysis of the ELS for the forces in the wall and support system.
6. Study of the effects of dewatering and excavation on the surroundings.
7. Connection details, dewatering or groundwater control scheme (if any).
8. Monitoring proposal for stress, deformation, water table *etc*.

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2. TYPES OF RETAINING SYSTEMS

Due to site constraints, ground and water table conditions, time and cost of construction, conditions of the adjoining buildings and other factors, different types of ELS have been used in Hong Kong. The basic wall types include:

1. **Sheet pile wall** – It is flexible and cheap with good water-tightness but is obstructed by utility. Virtually all sheet pile walls are temporary, though permanent walls have also been adopted in Hong Kong. A sheet pile wall is a relatively flexible system that is possibly the most common wall type in the world and is usually used for bottom-up construction. The Lok Fu MTR station in Hong Kong is a good example of sheet pile wall construction.
2. **Soldier pile wall** – It is usually a temporary wall that is used in stable ground, and the cost is usually low. Soldier pile walls usually achieve water-tightness but can avoid the disturbance of utility by allowing it to pass through the lagging. This system is usually for bottom-up construction. An interesting example is the soldier pile wall for the Diamond Hill MTR station in Hong Kong with hand-dug caisson as the permanent soldier pile.
3. **Caisson wall** – It is a system invented in Hong Kong. The wall can both be a temporary or permanent stiff wall, which still finds applications in China but not Hong Kong. Caisson wall construction requires less machines to avoid utility obstruction easily. This construction is dangerous to workers, with the loss of many lives in the past, and is seldom used currently. Robots have been developed for this construction recently, but the application of robot caisson construction is still very limited. The Caisson wall is used for both top-down and bottom-up construction, and some examples include the Sun Plaza and Choi Hung MTR station in Hong Kong.
4. **Diaphragm wall** – It is a popular stiff wall system in Hong Kong and many other countries and is mostly used as a usually permanent. It requires heavy machines in the construction and cannot avoid utility hence utility diversion is usually required. There are many examples, which include the Central, Wan Chai, Sheung Wan and Causeway Bay MTR stations, and Times Square in Hong Kong.
5. **Secant pile wall** – It is an expensive, stiff, permanent wall that requires heavy machines (RCD in Hong Kong) in construction. The construction of this wall may require relocation of utility, but it is a good water-tight wall; hence it is used for top-down construction in poor soil in Hong Kong. There are limited examples in Hong Kong, which include the Prince Edward and Mongkok MTR stations.

6. **Pipe pile wall** – It is a versatile, flexible and cheap temporary system that requires back grouting to achieve water-tightness. This wall is not common in other countries but is very common in Hong Kong. Similar to a soldier pile wall (actually pipe pile wall is a form of a soldier pile wall), it can avoid utility and is usually used for bottom up construction. Some examples include the Nathan Centre and the Hung Hom station in Hong Kong.
7. **PIP wall** – It is a form of small diameter bore pile wall (see Chapter 8) with normal grout pressure and the use of a special grout without aggregate. The reinforcement to the pile is usually a normal reinforcement cage, but steel H sections have also been used in Hong Kong. Some examples include the Tsim Sha Tsui and Jordan MTR stations and the Chartered Bank basement in Hong Kong. Due to the difficulty in good quality control and waterproofing, it is less common in recent times.
8. **Jet pile wall** – It is a wall formed by using a very high-pressure grout jetting and the grout pressure can reach 100 to 200 bars (double tube or triple tube technique in general). The structure of the soil can be destroyed under such a high pressure, the cement grout can hence mix well regardless of the permeability of the soil to form a good quality pile. This type of wall is used mainly in soft clay where the strength of clay is low with low permeability. Some examples include the MRT in Singapore and Sheung Wan MTR in Hong Kong.
9. **Mixed wall** – the combination of two or even three wall types may be adopted for special ground conditions. The most critical issue for mixed wall systems is the junctions between different wall types. A good example is the top-down Sun Plaza construction in Hong Kong, where diaphragm and caisson walls are used in different locations to deal with different ground conditions.

Deep excavation constructions have caused more problems as compared with other types of constructions, notably surrounding buildings affected by excessive wall and ground movement, difficulty in groundwater control, stability of bottom of the excavation, failure of a supporting system (mostly buckling of struts and failure of welding). The choice of a suitable retaining wall system is critical in terms of time and cost considerations, and the choice of excavation and support method and dewatering/recharge system is important in terms of the ground settlement and the effects on the surrounding buildings, particularly for ELS construction in urban areas. With reference to the ELS construction at Nathan Centre (Hong Kong), sheet pile wall is used in areas far away from the MTR tunnel for cost consideration, while pipe pile wall with low vibration is used in areas close to the Jordan MTR tunnel to reduce the induced vibration. For the construction of the Wan Chai and

CHAPTER 10

Illustrations of a Building Design, Foundation System, and Link Bridge

Abstract: This chapter introduces a complete design of the ELS, foundation, and superstructure in Hong Kong. The constraints and various considerations with respect to the final scheme for the foundation, ELS, and multi-storey buildings are discussed.

Keywords: Constraints, ELS, Foundation, Office block, Superstructure.

1. INTRODUCTION

To elaborate the materials that are covered in the previous 9 chapters, a sample project in Hong Kong as shown in Fig. (**10.1**) is chosen by the authors for illustration of the structural analysis and design of an office building in accordance with the local practice. The complete report and the corresponding input files and drawings can be obtained from the authors through a mail request at natureymc@yahoo.com.hk. The site of the proposed trapezoidal-shape office block is about 76 × 21 m^2 on plan. The proposed office block comprises 8 stories with 1 basement floor, which is connected to the nearby 2-storey Teaching Block by a link bridge. The building height of the office block is 31m approximately above the G/F. There will be a basement floor under the building, with a height of around 5m. A pump room should be installed there. Some constraints will also be imposed on the 1st floor and other floors. For the 1st floor, the E&M zone has to be cleared with no beams allowed and no columns permitted within the auditorium. A link bridge will connect the 3rd floor of the proposed office block to the teaching block. The roof of the link bridge will be covered by greenery to provide a better environment. In addition to the provision of means of access and circulation, the link bridge is also designed to serve as a canteen to provide catering services.

The surrounding screen walls are proposed from G/F to B/F to retain the lateral soil at the underground level. The design ground water table is assumed to be 2 m below the existing ground level for calculating resistance to buoyancy. Ground investigation indicates that there are mainly 4 types of soil which are Fill, Marine Deposits, Alluvium, and Completely Decomposed Granite in succession. The rock level is roughly 46m below the ground surface. The adjacent buildings are teaching blocks and already existing schools. So the choices of ELS and foundation designs are limited by noise pollution issues. The Mass Transit Railway (MTR) Tunnel is

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in close proximity with the site. Restrictions by the MTR requirements to construction projects nearby have to be observed which affects the piling layout design.

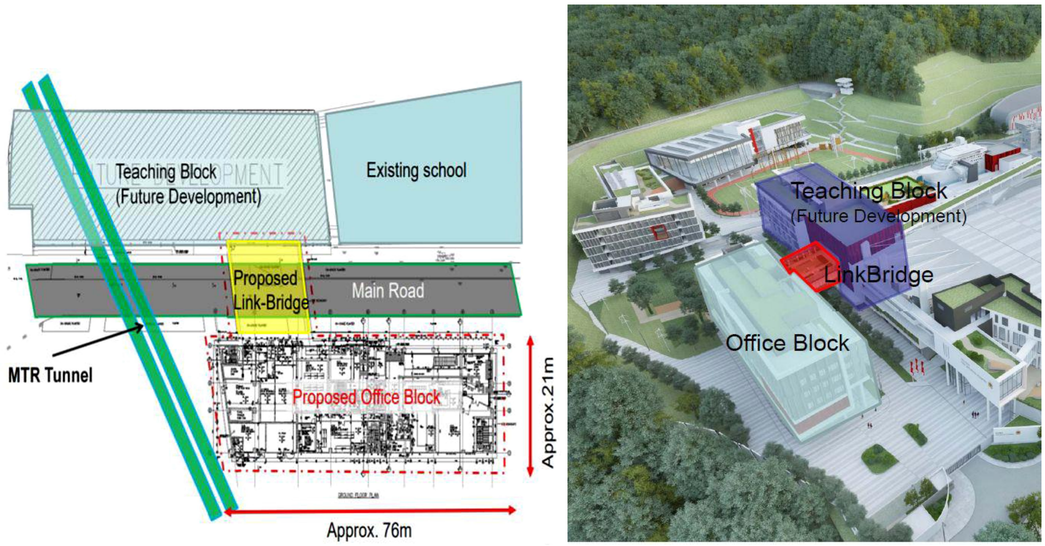


Fig. (10.1). Layout of the proposed development.

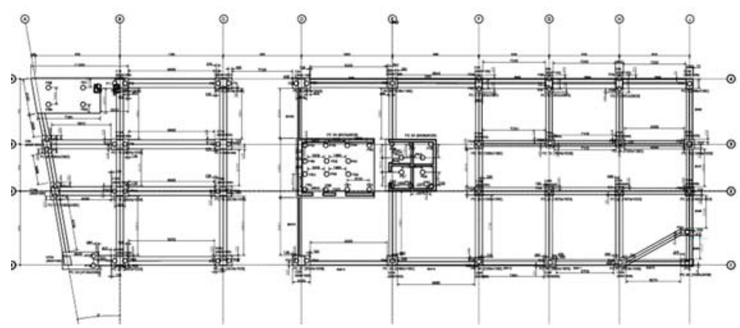
2. CHOICE OF FOUNDATION SCHEME

As the Teaching Block and the MTR tunnel are adjacent to the site, excessive noise and vibration should need to be minimized during construction.

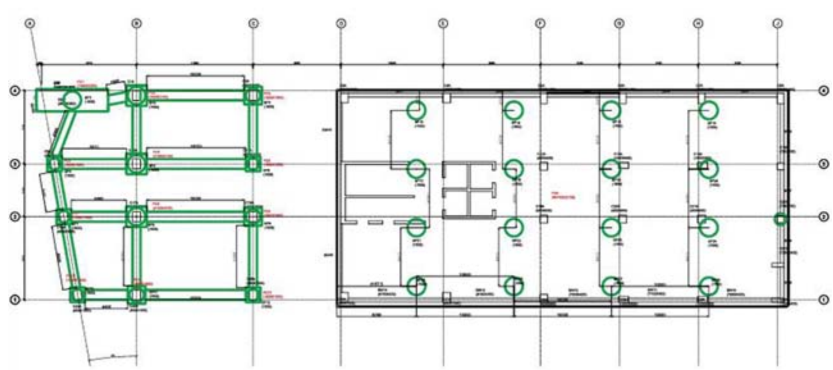
The construction methods must strictly follow the local practice imposed by the Practice Notes for Authorized Persons (PNAP) No. APP-24 which imposes the following restrictions:

1. Avoid piling works within 3m from the railway fence/wall or 7m from the centerline of the nearest railway track.
2. Minimize the vibration so that the vibration measured on the railway shall not exceed the peak particle velocity of 10mm/sec.
3. The effects of site formation or foundation works or excavation works shall not exceed the following limits: (1) The vertical or horizontal pressure shall not exceed 20 kPa, (2) Differential movement of railway track or plinth shall not exceed 20mm, (3) Induced level difference between rails of a track in perpendicular plane shall not exceed 5mm.

Pre-bored socket H-piles and large-diameter bored piles are preferred options for the project because less noise and vibration are generated during construction. The gravity loading from the superstructure is estimated by using the “tributary area method”. The capacity of each pile is calculated in accordance with the local Foundation Code 2017 and Cheng [1]. The number of piles is then found by the superstructure gravity loading divided by the pile capacity of a single pile. The piling layout plans for the two schemes are shown in Fig. (**10.2**). Feasibility of each scheme is assessed by checking the deflection and adequacy of pile load capacity. The lateral deflection is limited to 25mm in the local practice and the pile capacity is compared with the pile reactions calculated by the software SAFE/PLATE.



(a) Scheme 1 Pre-bored socket-H pile layout



(b) Scheme 2 Large diameter bore pile

Fig. (10.2). Two foundation schemes for the proposed development.

2.1. Choice of ELS System

The plan dimension of the excavation construction zone is 75.8 m length and 21.4 m width with 10m depth from +75 mPD to the proposed final excavation level. Sheet pile wall or a combination of sheet pile and pipe pile wall can be adopted for

APPENDIX A

The Newmark Method in Time History Analysis of Structure

A.1 Underlying Principle and Methodology of the Newmark Method

The dynamic motion of a structure of a single degree of freedom can be expressed in the matrix as

$$\boldsymbol{m\ddot{u}} + \boldsymbol{c\dot{u}} + \boldsymbol{ku} = \boldsymbol{F} \qquad \textbf{(A.1)}$$

where m is the mass of the structure, u is the displacement and $\dot{u}$ and $\ddot{u}$ are the first and second derivatives of u with time, respectively, which are physically velocity and acceleration; k is the stiffness of the offering a force to the structure with a magnitude proportional to the displacement and also in a direction opposing the displacement; c is similar to k but the magnitude is proportional to the velocity of the structure instead, which is commonly known as the damping coefficient.

Expanding (A.1) to a structure of multi-degree of freedom, the parameters become matrices instead of single values as:

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{Ku} = \boldsymbol{F} \qquad \textbf{(A.2)}$$

In (A.2), $\boldsymbol{M}$ is a mass matrix which is mostly a diagonal one with entries representing masses of the structure being mobilized to undergo motion varying with time; $\boldsymbol{C}$ is the damping matrix containing damping coefficients generally in the diagonal of the matrix. $\boldsymbol{K}$ is generally simply the stiffness matrix of the structure providing restoring force when the structure undergoes displacement $\boldsymbol{u}$**.** $\boldsymbol{\ddot{u}}$ and $\boldsymbol{\dot{u}}$ are the velocity and acceleration matrices, respectively.

When $\boldsymbol{\ddot{u}}$ and $\boldsymbol{\dot{u}}$ are taken as zero, (A.2) reduces to $\boldsymbol{Ku} = \boldsymbol{F}$ which is the static analysis of a structure.

To present $\boldsymbol{\ddot{u}}$ and $\boldsymbol{\dot{u}}$ in terms of $\boldsymbol{u}$ for possible solution of (A.2) by numerical method, Newmark (1959) has made use of Taylor's series listed as:

$$\boldsymbol{u_t} = \boldsymbol{u_{t-\Delta t}} + \Delta t\boldsymbol{\dot{u}_{t-\Delta t}} + \frac{\Delta t^2}{2}\boldsymbol{\ddot{u}_{t-\Delta t}} + \frac{\Delta t^3}{6}\boldsymbol{\dddot{u}_{t-\Delta t}} + \cdots \qquad \textbf{(A.3a)}$$

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$$\dot{\boldsymbol{u}}_{\boldsymbol{t}} = \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \Delta t \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \frac{\Delta t^2}{2} \dddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \cdots \qquad \textbf{(A.3b)}$$

In (A.3a) and (A.3b), Δt is a time step between successive "time stations" in which $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ are to be determined. Newmark truncated these equations by the use of two coefficients β and γ as:

$$\boldsymbol{u}_{\boldsymbol{t}} = \boldsymbol{u}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \Delta t \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \frac{\boldsymbol{\Delta t^2}}{2} \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \beta \Delta t^3 \dddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} \qquad \textbf{(A.3c)}$$

$$\dot{\boldsymbol{u}}_{\boldsymbol{t}} = \dot{\boldsymbol{u}}_{t-\Delta t} + \Delta t \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \gamma \Delta t^2 \dddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} \qquad \textbf{(A.3d)}$$

Newmark suggests $\gamma = 0.5$ to avoid artificial damping. For the coefficient β, if $\beta = 1/4$, it implies that the acceleration within the time interval Δt is constant. If $\beta = 1/6$, the acceleration within the time interval Δt is linearly varying.

If the acceleration is assumed to be linear within the time step,

$$\dddot{\boldsymbol{u}} = \frac{\ddot{\boldsymbol{u}}_{\boldsymbol{t}} - \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}}}{\Delta t} \qquad \textbf{(A.4)}$$

Substituting (A.4) into (A.3c) and (A.3d), Newmark's equations in the standard form are listed as:

$$\boldsymbol{u}_{\boldsymbol{t}} = \boldsymbol{u}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \Delta t \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \beta \Delta t^2 \ddot{\boldsymbol{u}}_{\boldsymbol{t}} \qquad \textbf{(A.5a)}$$

$$\dot{\boldsymbol{u}}_{\boldsymbol{t}} = \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + (1 - \gamma) \Delta t \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + \gamma \Delta t \ddot{\boldsymbol{u}}_{\boldsymbol{t}} \qquad \textbf{(A.5b)}$$

Wilson (1962) nevertheless formulated Newmark's method in matrix form (as in (A.2)). The need for iteration can also be eliminated by introducing the direct solution of equations at each time step. (A.5a) and (A.5b) can then be re-written as:

$$\ddot{\boldsymbol{u}}_{\boldsymbol{t}} = b_1(\boldsymbol{u}_{\boldsymbol{t}} - \boldsymbol{u}_{\boldsymbol{t}-\Delta \boldsymbol{t}}) + b_2 \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + b_3 \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} \qquad \textbf{(A.6a)}$$

$$\dot{\boldsymbol{u}}_{\boldsymbol{t}} = b_4(\boldsymbol{u}_{\boldsymbol{t}} - \boldsymbol{u}_{\boldsymbol{t}-\Delta \boldsymbol{t}}) + b_5 \dot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} + b_6 \ddot{\boldsymbol{u}}_{\boldsymbol{t}-\Delta \boldsymbol{t}} \qquad \textbf{(A.6b)}$$

Where the b_1 to b_6 coefficients as related to β and γ are as follows:

$$b_1 = \frac{1}{\beta \Delta t^2};\ b_2 = \frac{-1}{\beta \Delta t};\ b_3 = 1 - \frac{1}{2\beta};\ b_4 = \frac{\gamma}{\beta \Delta t};\ b_5 = 1 - \frac{\gamma}{\beta};\ b_6 = \left(1 - \frac{\gamma}{\beta}\right) \Delta t \quad \textbf{(A.7)}$$

Generally, $\gamma = 0.5$ and $\beta = 1/6$ for the assumption that acceleration is linearly varying within the time interval Δt. If $\beta = 0.25$, constant acceleration is assumed within the time interval. The latter possesses the advantage of being stable in computation regardless of the size of time step.

The substitution of (A.6a) and (A.6b) into (A.2) allows the dynamic equilibrium of the system at time "t" to be written in terms of the unknown node displacement $\boldsymbol{u_t}$**.** Or

$$(b_1\boldsymbol{M} + b_4\boldsymbol{C} + \boldsymbol{K})\boldsymbol{u_t} = \boldsymbol{F_t} + \boldsymbol{M}(b_1\boldsymbol{u_{t-\Delta t}} - b_2\dot{\boldsymbol{u}}_{\boldsymbol{t-\Delta t}} - b_3\ddot{\boldsymbol{u}}_{\boldsymbol{t-\Delta t}}) + \boldsymbol{C}(b_4\boldsymbol{u_{t-\Delta t}} - b_5\dot{\boldsymbol{u}}_{\boldsymbol{t-\Delta t}} - b_6\ddot{\boldsymbol{u}}_{\boldsymbol{t-\Delta t}}) \quad \textbf{(A.8)}$$

In (A.8) $(b_1\boldsymbol{M} + b_4\boldsymbol{C} + \boldsymbol{K})$is the effective stiffness matrix $\overline{\boldsymbol{K}}$

So the procedure for solution can be as follows:

(i) Form the mass matrix $\boldsymbol{M}$, damping matrix $\boldsymbol{C}$ and Stiffness matrix $\boldsymbol{K}$. The stiffness matrix $\boldsymbol{K}$ is formed in the usual manner as for static analysis. For $\boldsymbol{M}$, and $\boldsymbol{C}$**,** they should be in the diagonal of the matrices where the degrees of freedom are to be studied. If only the lateral movement is interested, only the arrays corresponding to the lateral displacement require input.

$$\begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \end{bmatrix}$$

$\boldsymbol{C}$ is similarly formed.

(ii) Choose the values β and γ and calculate the b-coefficients under (A.7).

(iii) Form the effective matrix $\boldsymbol{K_{eff}} = \boldsymbol{K} + b_1\boldsymbol{M} + b_4\boldsymbol{C}$.

(iv) Specify the initial conditions for $\boldsymbol{u}$**,** $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ at time $t = 0 = t_0$ as $\boldsymbol{u_0}$**,** $\dot{\boldsymbol{u}}_{\boldsymbol{0}}$ **and** $\ddot{\boldsymbol{u}}_{\boldsymbol{0}}$.

(v) Choose a value for time interval Δt, say 0.05 to 0.1 sec.

(vi) Calculate the effective load vector at the first-time increment $t_1 = t_0 + \Delta t$ (then $t_0 = t_1 - \Delta t$) as:

$$\boldsymbol{F}_{\boldsymbol{eff@t_1}} = \boldsymbol{F}(t_1) + \boldsymbol{M}(b_1\boldsymbol{u_0} - b_2\dot{\boldsymbol{u}}_{\boldsymbol{0}} - b_3\ddot{\boldsymbol{u}}_{\boldsymbol{0}}) + \boldsymbol{C}(b_4\boldsymbol{u_0} - b_5\dot{\boldsymbol{u}}_{\boldsymbol{0}} - b_6\ddot{\boldsymbol{u}}_{\boldsymbol{0}})$$

Where $\boldsymbol{F}(t_1)$ is the externally applied force which is time dependent.

(vii) Solve the Node displacement $\boldsymbol{u_1}$ by $\boldsymbol{u_1} = \boldsymbol{K_{eff}}^{-1}\boldsymbol{F}_{\boldsymbol{eff@t_1}}$.

(viii) Calculate the node velocities and accelerations at t_1 by:

$$\dot{\boldsymbol{u}}_{\boldsymbol{1}} = b_4(\boldsymbol{u_1} - \boldsymbol{u_0}) + b_5\dot{\boldsymbol{u}}_{\boldsymbol{0}} + b_6\ddot{\boldsymbol{u}}_{\boldsymbol{0}}$$
$$\ddot{\boldsymbol{u}}_{\boldsymbol{1}} = b_1(\boldsymbol{u_1} - \boldsymbol{u_0}) + b_2\dot{\boldsymbol{u}}_{\boldsymbol{0}} + b_3\ddot{\boldsymbol{u}}_{\boldsymbol{0}}$$

(ix) Then repeat step (vi) with $\boldsymbol{u_1}$ as the previous $\boldsymbol{u_0}$ to calculate $\boldsymbol{u_1}$ and so on until the time the designer wishes to stop the calculations. For convenience, the equations in (vi) to (viii) are relisted in general terms of t_i as the ith time station:

$$\boldsymbol{F}_{\boldsymbol{eff(t_i)}} = \boldsymbol{F}(t_i) + \boldsymbol{M}\left(b_1\boldsymbol{u}_{\boldsymbol{t_i}-\Delta t} - b_2\dot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t} - b_3\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t}\right) + \boldsymbol{C}\Big(b_4\boldsymbol{u}_{\boldsymbol{t_i}-\Delta t} - b_5\dot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t} - b_6\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t}\Big)$$

$$\boldsymbol{u}_{\boldsymbol{t_i}} = \boldsymbol{K_{eff}}^{-1}\boldsymbol{F}_{\boldsymbol{eff(t_i)}}$$
$$\dot{\boldsymbol{u}}_{\boldsymbol{t_i}} = b_4\left(\boldsymbol{u}_{\boldsymbol{t_i}} - \boldsymbol{u}_{\boldsymbol{t_i}-\Delta t}\right) + b_5\dot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t} + b_6\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t}$$
$$\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}} = b_1\left(\boldsymbol{u}_{\boldsymbol{t_i}} - \boldsymbol{u}_{\boldsymbol{0t_i}-\Delta t}\right) + b_2\dot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t} + b_3\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}-\Delta t}$$

(x) $\boldsymbol{u}$ can still be calculated for certain intervals even the externally applied force $\boldsymbol{F}(t_i)$ stops to apply by simply setting $\boldsymbol{F}(t_i) = 0$.

(xi) Upon acquisition of the deflections, the internal forces of the structure can be determined by multiplying the individual members' stiffness to their node displacements.

A.2 Worked Example of 3 storey Plane Frame

A plane frame of structural sizes and masses on the beams as shown in Fig. (**A.1**) is to be analyzed to the base acceleration of $\ddot{u}_g(t) = 1 \times \sin(2.5\pi t)$, which is a sine function of frequency 1.25Hz or period $1/1.25 = 0.8$ sec. The analysis is to be executed for 5 sec. 5% damping is assumed. Two cases are to be performed: (1) $\ddot{u}_g(t)$ to be applied throughout the 5 sec; and (2) $\ddot{u}_g(t)$ only for the first 2 sec.

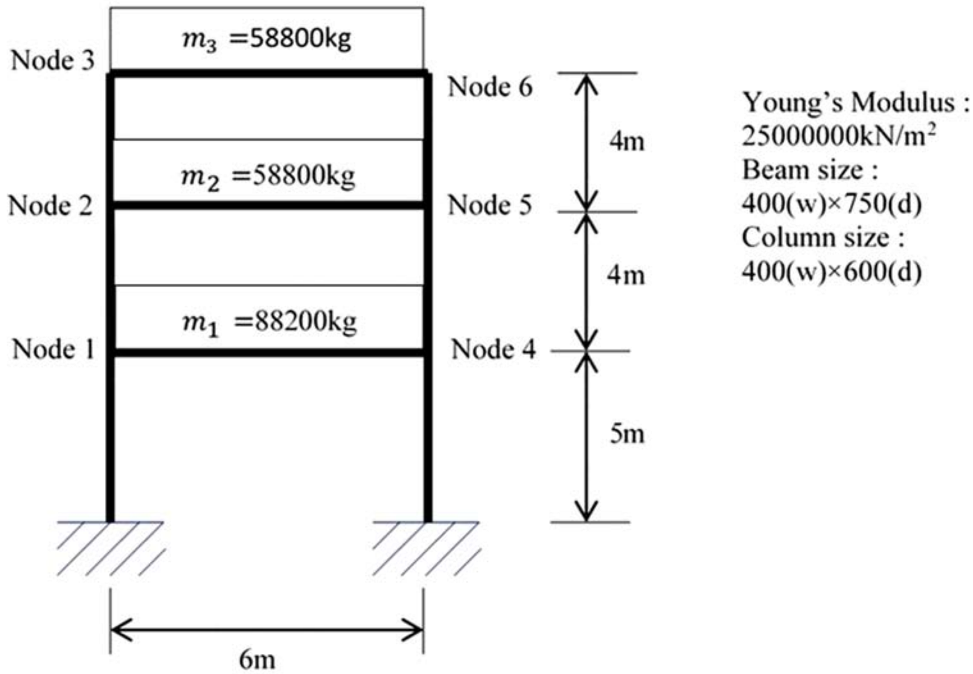

Fig. (A.1). Portal frame for study of structure movement due to ground acceleration.

(i) The structure is one with 18 degrees of freedom after eliminating the fixed support conditions. The stiffness matrix $\boldsymbol{K}$ is formed in the usual manner. As the analysis is confined to lateral displacement, in the formulation of the mass matrix $\boldsymbol{M}$, the masses are split as shown in Fig. (**3.23**) and added to the entries corresponding to the lateral displacement. So $\boldsymbol{M}$ and the corresponding $\boldsymbol{u}$ matrix are listed as follows. In the $\boldsymbol{u}$ matrix, u stands for horizontal translation, v stands for vertical translation and θ stands for rotation.

$$\boldsymbol{M}=\begin{bmatrix}
44.1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&29.4&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&29.4&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&44.1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&29.4&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&29.4&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

$$\boldsymbol{u}=\begin{bmatrix}u_1\\v_1\\\theta_1\\u_2\\v_2\\\theta_2\\u_3\\v_3\\\theta_3\\u_4\\v_4\\\theta_4\\u_5\\v_5\\\theta_5\\u_6\\v_6\\\theta_6\end{bmatrix}$$

The natural frequencies of the structure are first to be found. It is the eigenvalue problem of the matrix $\boldsymbol{K}^{-1}\boldsymbol{M}$. As $\boldsymbol{M}$ contains only masses in the horizontal translation, the natural frequencies are only related to the horizontal translation. The lowest 3 natural frequencies corresponding to the greatest eigenvalues λ ($=1/\omega^2=1/(2\pi f)^2$ where f is the natural frequency of the structure) are (1) $\lambda_1=0.012924$; (2) $\lambda_2=0.0014404$; (3) $\lambda_3=0.000377103$. The periods and frequencies are $(f = 1/(2\pi\sqrt{\lambda})$ and $T = 1/f)$ (1) $f_1 = 1.399980$Hz, $T_1 = 0.714296$sec; (2) $f_2 = 4.194352$Hz, $T_2 = 0.238463$sec; (3), $f_3 = 8.195781$Hz, $T_3 = 0.122014$sec.

For conservative design, the lowest circular frequency $\omega_1 = 2\pi f_1 = 8.7963$ is used to calculate the damping coefficients as $\zeta 2m\omega$ where ζ is the damping ratio taken as 0.05. So the damping coefficients for the 6 point masses at the three levels are:

$C_1 = 0.05 \times 2 \times 44.1 \times 8.7963 = 38.792$kNsec/m (Storey 1)

$C_2 = C_3 = 0.05 \times 2 \times 29.8 \times 8.7963 = 26.213$kNsec/m (Storey 1 & 2)

The damping matrix $\boldsymbol{C}$ can then be formed in the similar manner as $\boldsymbol{M}$ with the mass entries replaced by the damping coefficients.

(ii) γ is taken as 0.5, β is taken as 0.25 for assuming constant acceleration within the time interval Δt (taken as 0.05sec). So b coefficients are calculated as follows:

$b_1 = \frac{1}{\beta \Delta t^2} = 1600; b_2 = \frac{-1}{\beta \Delta t} = -80; b_3 = 1 - \frac{1}{2\beta} = -1;$

$b_4 = \frac{\gamma}{\beta \Delta t} = 40; b_5 = 1 - \frac{\gamma}{\beta} = -1;\ b_6 = \left(1 - \frac{\gamma}{\beta}\right)\Delta t = 0$

(iii) The effective matrix $\boldsymbol{K_{eff}} = \boldsymbol{K} + b_1\boldsymbol{M} + b_4\boldsymbol{C}$ is formed;

(iv) The initial values of $\boldsymbol{u}, \dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ at time $t = 0 = t_0$ as $\boldsymbol{u_0}, \dot{\boldsymbol{u}}_\mathbf{0}$ **and** $\ddot{\boldsymbol{u}}_\mathbf{0}$ are all set to 0;

(v) Δt is taken as 0.05sec;

(vi) The ground acceleration as the external excitation to the frame is taken as $\ddot{u}_g(t) = -1\sin(2.5\pi t)$ with a maximum value at 1.0m/sec^2. It should be noted that negative acceleration by the sine function implies positive movement. The effective load vector at the first time increment $t_1 = t_0 + \Delta t = 0 + 0.05$ is calculated as:

$$\boldsymbol{F_{eff@t_1}} = \boldsymbol{F}(t_1) + \boldsymbol{M}(b_1\boldsymbol{u_0} - b_2\dot{\boldsymbol{u}}_\mathbf{0} - b_3\ddot{\boldsymbol{u}}_\mathbf{0}) + \boldsymbol{C}(b_4\boldsymbol{u_0} - b_5\dot{\boldsymbol{u}}_\mathbf{0} - b_6\ddot{\boldsymbol{u}}_\mathbf{0})$$

where $\boldsymbol{F}(t_1) = \ddot{u}_g(t_1)\boldsymbol{M}.\boldsymbol{U_c} = \boldsymbol{M}\sin(2.5\pi t_1)$ is the externally applied force where $\boldsymbol{U_c}$ is a column matrix of array size 18×1 with all entries equal to 1. The purpose is to convert $\ddot{u}_g(t_1)\boldsymbol{M}$ from a square matrix into column matrix.

(vii) The Node displacement $\boldsymbol{u_1}$ is solved by $\boldsymbol{u_1} = \boldsymbol{K_{eff}}^{-1}\boldsymbol{F_{eff@t_1}}$, which contains horizontal, vertical translations and rotations of the nodes;

(viii) The node velocities and accelerations at t_1 are calculated as,

$$\dot{\boldsymbol{u}}_{\boldsymbol{1}} = b_4(\boldsymbol{u_1} - \boldsymbol{u_0}) + b_5\dot{\boldsymbol{u}}_{\boldsymbol{0}} + b_6\ddot{\boldsymbol{u}}_{\boldsymbol{0}}$$

$$\ddot{\boldsymbol{u}}_{\boldsymbol{1}} = b_1(\boldsymbol{u_1} - \boldsymbol{u_0}) + b_2\dot{\boldsymbol{u}}_{\boldsymbol{0}} + b_3\ddot{\boldsymbol{u}}_{\boldsymbol{0}}$$

(ix) With the $\boldsymbol{u_1}$, $\dot{\boldsymbol{u}}_{\boldsymbol{1}}$ and $\ddot{\boldsymbol{u}}_{\boldsymbol{1}}$ obtained, the effective load vector is calculated as in step (vi) for $t_2 = t_1 + \Delta t = 0.05 + 0.05 = 0.1$sec and then $\boldsymbol{u_2}$, $\dot{\boldsymbol{u}}_{\boldsymbol{2}}$ and $\ddot{\boldsymbol{u}}_{\boldsymbol{2}}$. The process is re-current by the following general formulae

$$\boldsymbol{F_{eff(t_i)}} = \boldsymbol{F}(t_i) + \boldsymbol{M}\left(b_1\boldsymbol{u_{t_i-\Delta t}} - b_2\dot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}} - b_3\ddot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}}\right) + \boldsymbol{C}\left(b_4\boldsymbol{u_{t_i-\Delta t}} - b_5\dot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}} - b_6\ddot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}}\right)$$

$$\boldsymbol{u_{t_i}} = \boldsymbol{K_{eff}}^{-1}\boldsymbol{F_{eff(t_i)}}$$

$$\dot{\boldsymbol{u}}_{\boldsymbol{t_i}} = b_4\left(\boldsymbol{u_{t_i}} - \boldsymbol{u_{t_i-\Delta t}}\right) + b_5\dot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}} + b_6\ddot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}}$$

$$\ddot{\boldsymbol{u}}_{\boldsymbol{t_i}} = b_1\left(\boldsymbol{u_{t_i}} - \boldsymbol{u_{0t_i-\Delta t}}\right) + b_2\dot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}} + b_3\ddot{\boldsymbol{u}}_{\boldsymbol{t_i-\Delta t}}$$

For the first analysis, the external ground acceleration persists for 5 seconds during which $\boldsymbol{F}(t_i)$ applies and the structure motions are analyzed.

(x) For the second case of analysis, $\boldsymbol{F}(t_i)$ is set to 2sec whilst the structure is then left to vibrate freely.

With the results of the analysis, the horizontal displacements (relative to the ground as per discussion in 3.6.2) at the storey levels are extracted and plotted, together with the external excitation in terms of movement which is obtained by $\int\int \ddot{u}_g(t)dtdt = \int\int -1 \times \sin(2.5\pi t)\,dtdt = 0.0162\sin(2.5\pi t)$. Fig. (**A.2**) contains the plot where the external excitation sustains throughout. It can be seen that the storey level lateral displacements increase with time even under damping. However, in Fig. (**A.3**) showing the structure movement where the excitation only persists for the first 2 sec, the lateral displacement starts to diminish with time after $t = 2$.

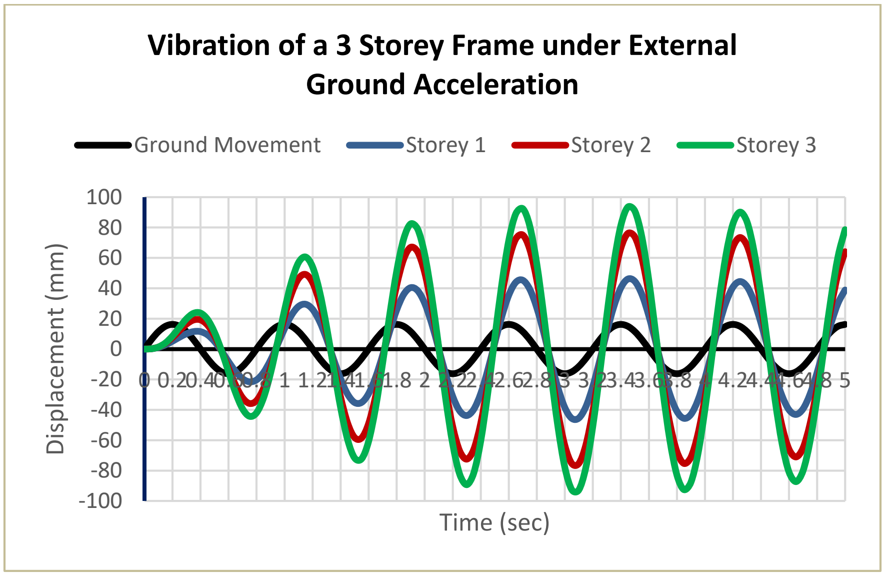

Fig. (A.2). Movement of Frame (Relative to Ground) in Fig (**A.1**) under Continuous External Ground Excitation.

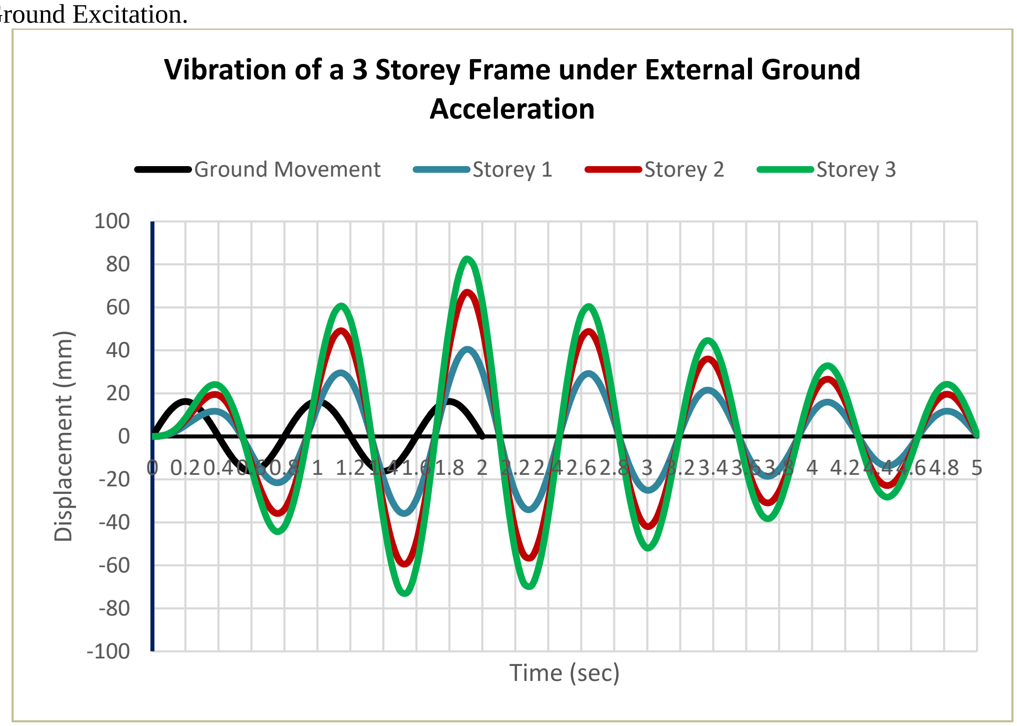

Fig. (A.3). Movement of Frame (Relative to Ground) in Fig. (**A.1**) under External Ground Excitation Persisting Only in the First 2 sec.

From Fig. (**A.2**), it can be seen that the external excitation has quite significant effect on the storey vibrations which increases with time. This can be explained by the closeness between the natural frequency of the structure at 1.4Hz (under para. A2 (i)) and the external excitation at 1.25Hz which is close to resonance. In addition, the structure vibrates with the external excitation frequency when the external excitation applies. However, when the external excitation ceases, it resume its natural frequency vibration.

The horizontal accelerations *vs* time as shown in Fig. (**A.4**) which is the seismic record of El Centro earthquake for the first 10 seconds are taken for analysis of the three-storey frame by the Newmark approach.

In the analysis, the $\ddot{u}_g(t_i)$ in step (vi) is taken equal to the acceleration in Fig. (**A.4**). The time step size is taken to be 0.02 sec which is also the time step size of Fig. (**A.4**).

The analytical result of the storey displacement of the frame (relative to the ground) is shown in Fig. (**A.5**).

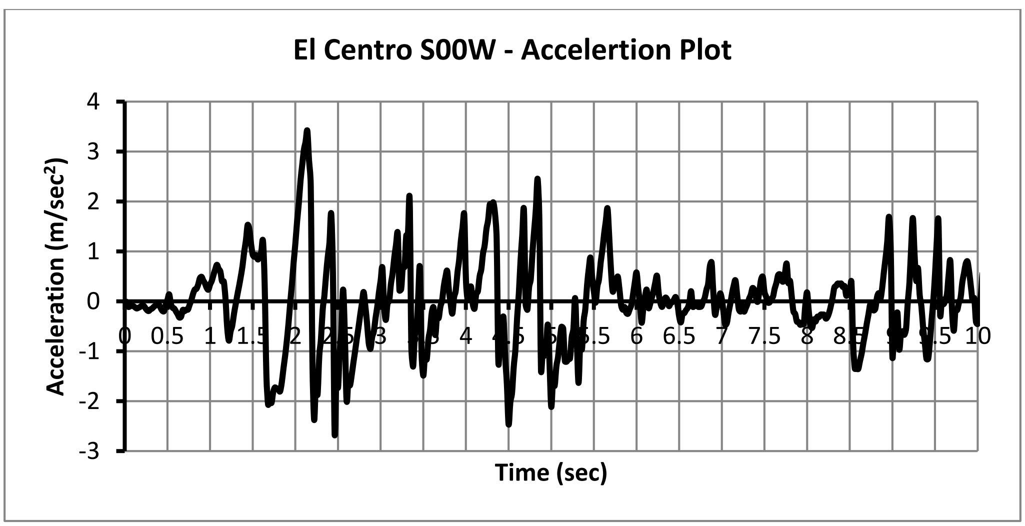

Fig. (A.4). Seismic Record of El Centro S00E for the first 10 second.

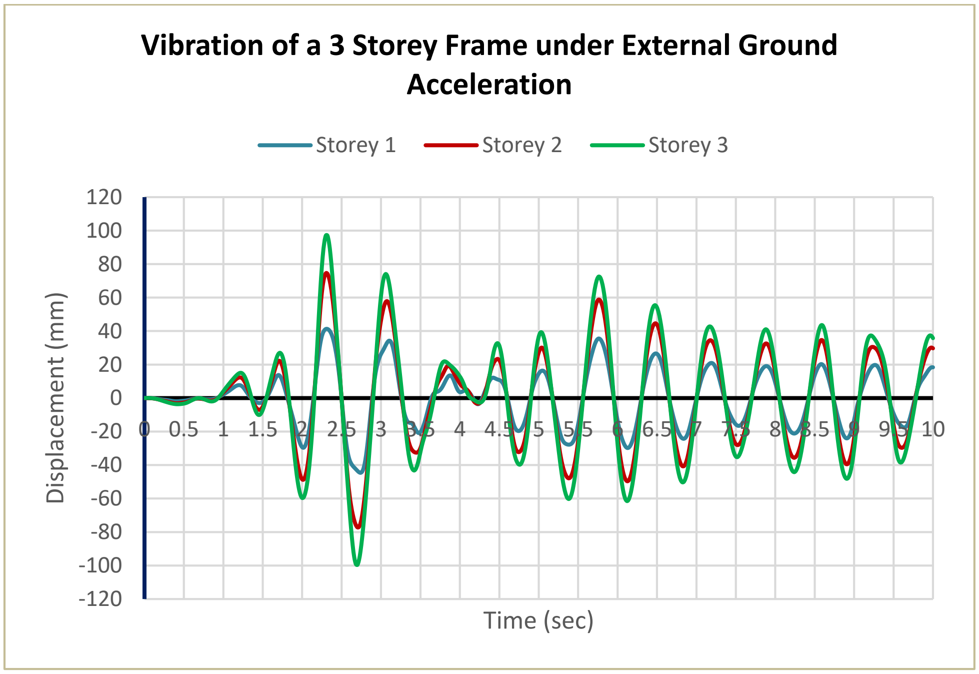

Fig. (A.5). Movement of Frame (relative to the ground) under El Centro Seismic S00E for the first 10 second.

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APPENDIX B

A Worked Example of Determination of Lateral Seismic Load in Accordance with the Response Spectrum Method

Consider Fig. (**B.1**), 3-storey 1-bay plane frame which is identical to that used in Appendix A for demonstration of time history analysis with ground movements.

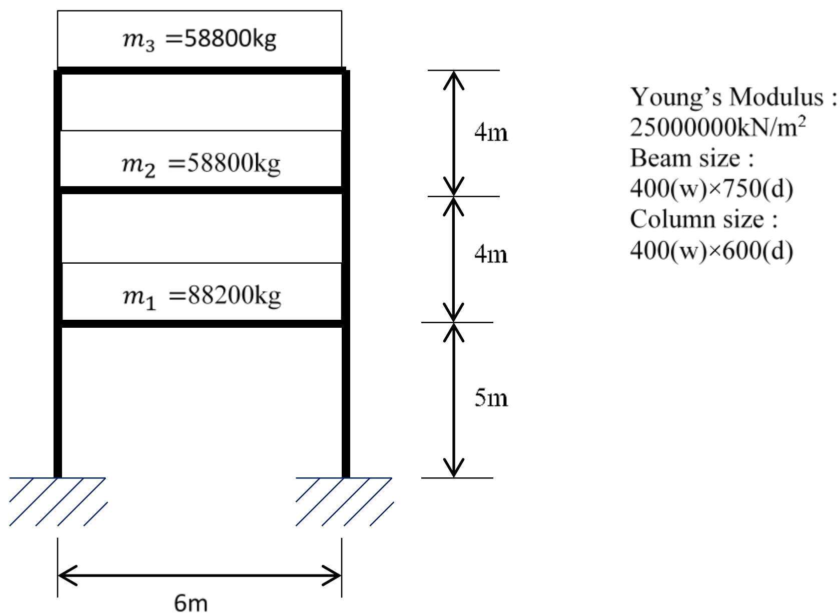

Fig. (B.1). Portal frame for determination of seismic load to response spectrum method.

The following procedures are adopted:

(i) Find the structure's fundamental natural periods:

The natural frequencies of the structure have been found in Appendix A.

After analysis, the first three highest eigenvalues with eigenvectors showing movements in the lateral directions are: (1) $\lambda_1 = 0.012924$; (2) $\lambda_2 = 0.0014404$;

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(3) $\lambda_3 = 0.000377103$. The periods are ($\mathrm{T} = 2\pi\sqrt{\lambda}$) (1) $T_1 = 0.714296$sec; (2) $T_2 = 0.238463$sec; (3) $T_3 = 0.122014$sec

The corresponding eigenvectors which are the vibration mode shapes of the fundamental frequencies are:

$$\phi_1 = \begin{bmatrix} 0.248 \\ 0.415 \\ 0.514 \end{bmatrix}; \phi_2 = \begin{bmatrix} -0.502 \\ -0.129 \\ 0.466 \end{bmatrix}; \phi_3 = \begin{bmatrix} 0.269 \\ -0.579 \\ 0.272 \end{bmatrix}$$

The mode shapes are plotted as follows (Fig. **B.2**):

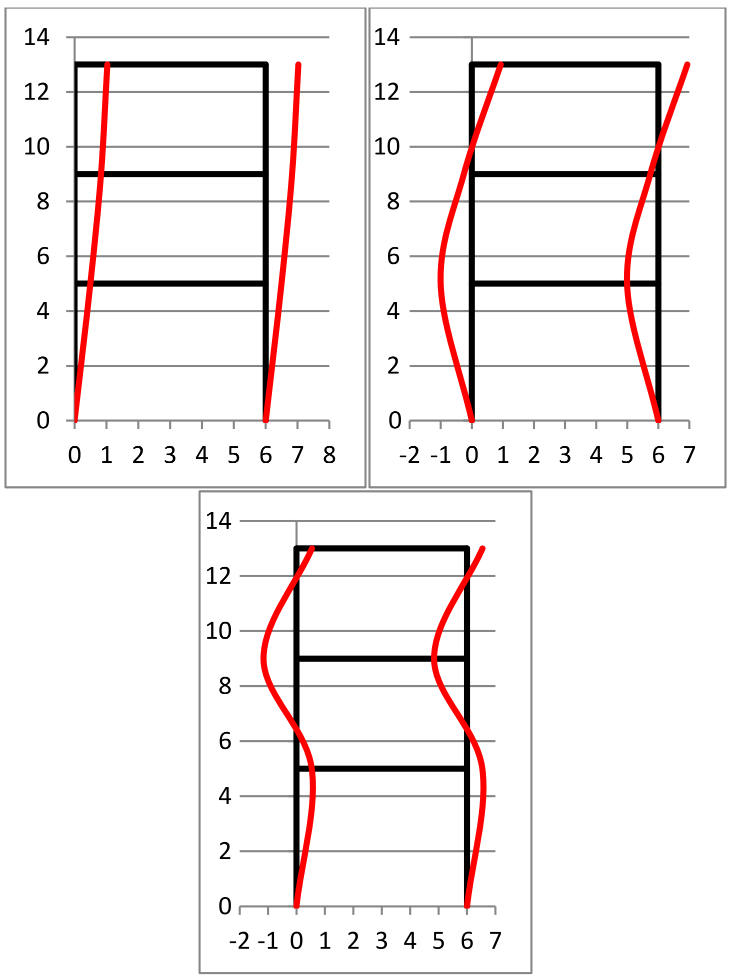

Fig. (B.2). Deflection modes of the highest three periods.

The sum of the modal mass participation of the 3 modes is equal to the sum of the masses $(58800 \times 2 + 88200 = 205800$kg. So the three modes are used.

(ii) Selection of a response spectrum:

The response spectrum from ASCE7-16 is chosen with the followings parameters:

$S_{DS} = 0.222g$; $S_{D1} = 0.08g$;$T_S = 0.36$sec;$T_0 = 0.07$sec; $T_L = 6$sec.

So the response spectrum is constructed (Fig. **B.2**).

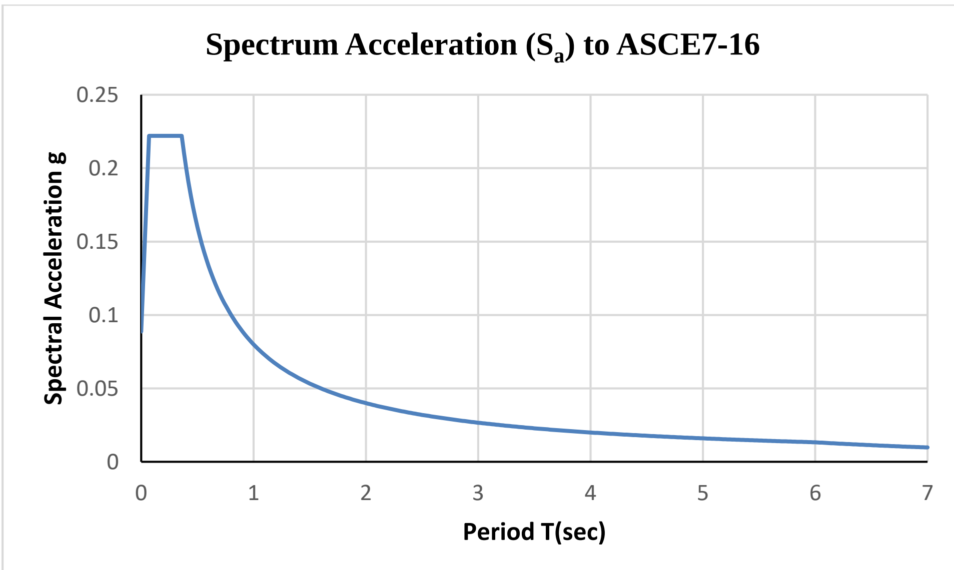

Fig. (B.3). Response spectrum to be used in the worked example.

The spectrum acceleration of the 3 modes based on their periods as read from Fig. (**B.3**) are (1) $S_{a1} = 0.112$g; (2) $S_{a2} = 0.222$g; (3) $S_{a3} = 0.222$g.

(iii) The modal acceleration factors of the 3 modes are calculated as follows:

Mode 1

Storey	M	ϕ	$M\phi$	$M\phi^2$	$\Gamma_1 S_{a1} M\phi$
1	88.2	0.248	21.8736	5.4246528	59.14088198

2	58.8	0.415	24.402	10.12683	65.9770592
3	58.8	0.514	30.2232	15.534725	81.71616488
		Sum	76.4988	31.086208	

The modal participation factor is $\Gamma_1 = 31.086208/76.4988 = 2.4608$

Mode 2

Storey	M	ϕ	$M\phi$	$M\phi^2$	$\Gamma_2 S_{a2} M\phi$
1	88.2	-0.502	-44.2764	22.226753	65.56559587
2	58.8	-0.129	-7.5852	0.9784908	11.23235308
3	58.8	0.466	27.4008	12.768773	-40.57578709
		Sum	-24.4608	35.974016	

The modal participation factor is $\Gamma_2 = 35.974016/-24.4608 = -0.67996$

Mode 3

Storey	M	ϕ	$M\phi$	$M\phi^2$	$\Gamma_3 S_{a3} M\phi$
1	88.2	0.269	23.7258	6.3822402	9.630220089
2	58.8	-0.579	-34.0452	19.712171	-13.81882883
3	58.8	0.272	15.9936	4.3502592	6.491746875
		Sum	5.6742	30.44467	

The modal participation factor is $\Gamma_3 = 30.44467/5.6742 = 0.18638$

It should be noted that the Γ_s calculated will be used to calculate the seismic shear force of the mode at the storeys in the last column for the 3 modes in the above tables.

(iv) The final shears on the storeys are summed by the SRSS method for the 3 modes as:

(1) Storey 1 $\sqrt{59.141^2 + 65.566^2 + (-9.63)^2} = 88.821\text{kN}$

(2) Storey 2 $\sqrt{65.977^2 + 11.232^2 + (-13.819)^2} = 63.338\text{kN}$

(3) Storey 3 $\sqrt{81.716^2 + (-40.576)^2 + 4.350^2} = 91.466\text{kN}$

The final shears will be applied to the structural as static loads for analysis to account for the seismic effects.

APPENDIX C

Moment Distribution and Portal Frame Analysis in Buildings in the Pre-Computer Era

C.1 Moment distribution method for continuous beam analysis in buildings

1. In the pre-computer era, the moment distribution method developed by Hardy Cross (1930) was the most (or the only) practical method in continuous beam analysis (to the elastic theory) as other accurate methods involve solutions of simultaneous equations of many unknowns which are difficult, if not impossible to handle without a computer;
2. Theoretically, the moment distribution method can give very accurate results by executing many cycles of distribution, though practically the user may execute a few to arrive at the accuracy he is aiming at. This "exact" method is taught in undergraduate courses;
3. However, as similar to other methods of analysis, continuous beam analysis of the applied loads involves many load cases as there are many arrangements of live loads on the various spans of the beam. So an approximate method has been formulated that requires only "one" cycle of distribution by the loads on a span. By a convenient format, the user can easily sort out the effects from the spans that can give the most critical values;
4. The approximate method has been widely used in the pre-computer era, 1960s to 1970s which is demonstrated in detail as follows through a worked example.

A worked example showing an analysis of the continuous Beam B1-B2-B3 with the employment of the sub-frame theory is presented as in Figs. (**C.1**) and (**C.2**).

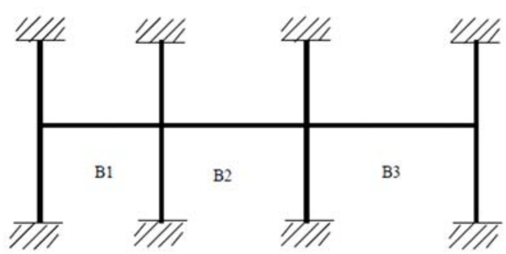

Fig. (C.1). Sub-frame for the Beam B1-B2-B3.

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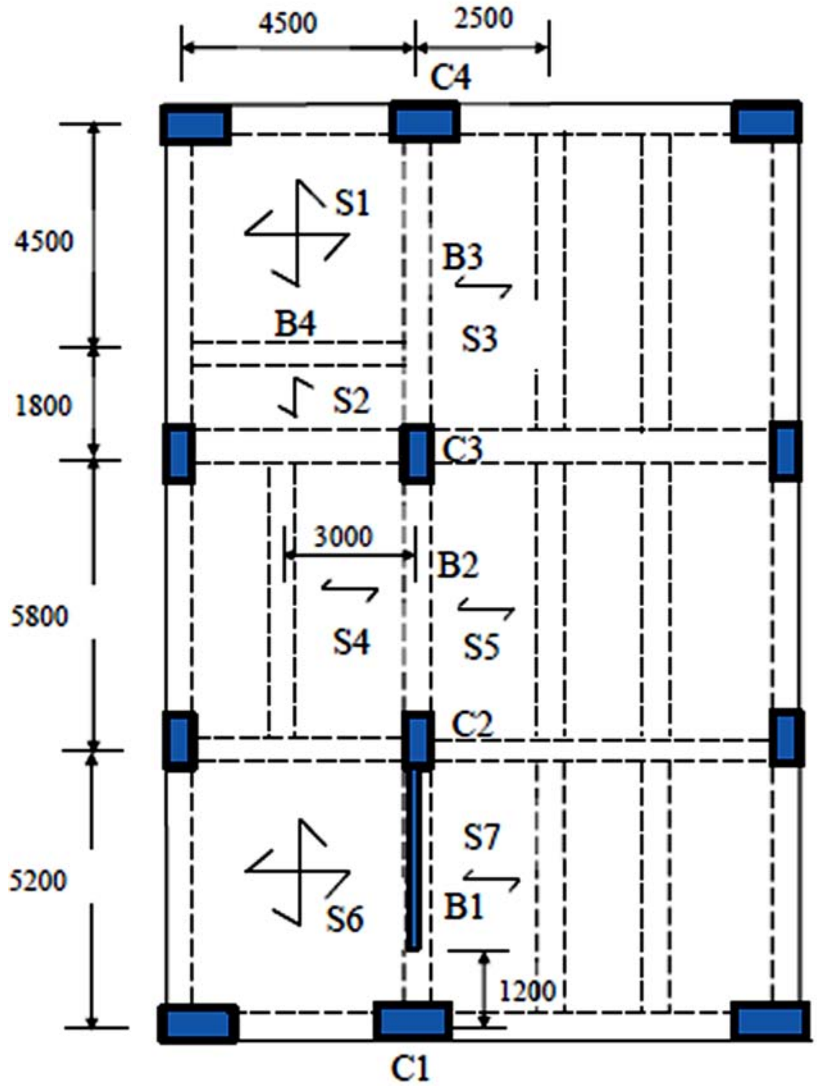


Fig. (C.2). Framing plan indicating Beam B1-B2-B3.

All slabs are 150mm thick.

B1 to B3 are 500(D) × 400 (W)

B4 is 450(D) × 350 (W)

Loads of Slabs

DL OW $0.15 \times 24.5 = 3.675$ kN/m^2

Fin 1.2 kN/m^2

4.875 kN/m^2

LL 3.0 kN/m^2

Loads from B4s

B4

		Description	L SH	R SH	L FEM	R FEM
B1, span = 5.2	D	OW $0.4\times0.45\times5.2\times24.5=22.932$ S7 $1.25\times5.2\times4.875=\underline{31.688}$ 54.620	27.31	27.31	23.669	23.669
		Parti $0.15\times2.5\times4\times20=30$	11.538	18.462	13.018	16.213
		S6 $0.5(5.2+0.7)\times2.25\times4.875=32.358$	16.179	16.179	17.464	17.464
		sum	55.027	61.951	54.150	57.345
	L	S7 $1.25\times5.2\times3.0=19.5$	9.750	9.750	8.450	8.450
		S6 $0.5(5.2+0.7)\times2.25\times3.0=19.913$	9.957	9.957	10.747	10.747
		sum	19.707	19.707	19.197	19.197
B2, span = 5.8m	D	OW $0.4\times0.45\times5.8\times24.5=25.578$ S5 $1.25\times5.8\times4.875=35.344$ S4 $1.5\times5.8\times4.875=\underline{42.413}$ 103.335	51.668	51.668	49.945	49.945
		sum	51.668	51.688	49.945	49.945
	L	S5 $1.25\times5.8\times3.0=21.75$ S4 $1.5\times5.8\times3.0=\underline{26.10}$ 47.85	23.925	23.925	23.128	23.128

		sum	23.925	23.925	23.128	23.128
B3, Span = 6.3m	D	OW $0.4\times0.45\times6.3\times24.5=27.783$ S3 $1.25\times6.3\times4.875=\underline{38.391}$ 66.174	33.087	33.087	34.741	34.741
		B4 28.00	20.000	8.000	25.714	10.286
		S1 $0.25\times4.5^2\times4.875=24.68$	8.814	15.866	12.513	19.879
		sum	61.901	56.953	69.727	61.665
	L	S3 $1.25\times6.3\times3.0=23.625$	11.813	11.813	12.403	12.403
		B4 13.699	9.785	3.914	12.581	5.032
		S1 $0.25\times4.5^2\times3.0=15.188$	5.424	9.764	7.701	12.234
		sum	27.022	25.490	32.684	29.669

DL OW $0.35 \times 0.3 \times 4.5 \times 24.5 =$ 11.576kN

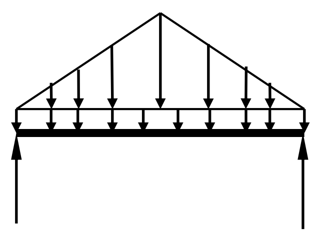
S1 $4.5^2/4 \times 4.875 = 24.68\text{kN}$

S2 $1.8/2\times 4.5 \times 4.875 = 19.744\text{kN}$

56.00kN

LL S1 $4.5^2/4 \times 3.0 = 15.188\text{kN}$

S2 $1.8/2\times 4.5 \times 3 = 12.15\text{kN}$



DL : 56/2
=28kN
LL : 27.338/2
=13.669kN

Loading Diagram

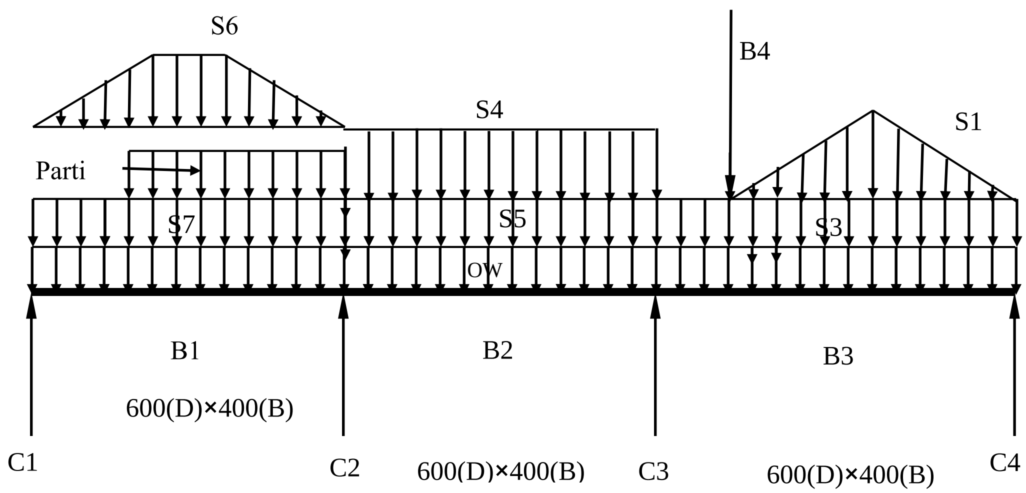


Fig. (C.3). Loading Diagram on Beam B1-B2-B3.

Beam Stiffness

B1 : 500×400 $$I_{B1} = \frac{bd^3}{12} = \frac{0.4\times0.6^3}{12} = 0.0072\text{m}^4$$

$$K_{B1} = \frac{4EI}{L} = \frac{4\times25100000\times0.0072}{5.2} = 139015\text{kNm}$$

B2 : 500×400 $$I_{B2} = I_{B1} = 0.0072\text{m}^4$$

$$K_{B2} = \frac{4EI}{L} = \frac{4\times25100000\times0.0072}{5.8} = 124634\text{kNm};$$

B3 : 500×400 $$I_{B3} = I_{B1} = 0.0072\text{m}^4$$

$$K_{B3} = \frac{4EI}{L} = \frac{4\times25100000\times0.0072}{63} = 114743\text{kNm}$$

Column Stiffness (Height = 3m)

C1 : 800×450 $$I_{C1} = \frac{bd^3}{12} = \frac{0.8\times0.45^3}{12} = 0.006075\text{m}^4$$

$$K_{C1} = \frac{4EI}{L} = \frac{4\times26400000\times0.006075}{3} = 427680\text{kNm}$$

C2 : 400×600 $I_{C2} = \frac{bd^3}{12} = \frac{0.4 \times 0.6^3}{12} = 0.0072\text{m}^4$

$$K_{C2} = \frac{4EI}{L} = \frac{4 \times 26400000 \times 0.0072}{3} = 506880\text{kNm}$$

C3 : 400×600 $I_{C3} = \frac{bd^3}{12} = \frac{0.4 \times 0.6^3}{12} = 0.0072\text{m}^4$

$$K_{C3} = \frac{4EI}{L} = \frac{4 \times 26400000 \times 0.0072}{3} = 506880\text{kNm}$$

C4 : 800×450 $I_{C4} = \frac{bd^3}{12} = \frac{0.8 \times 0.45^3}{12} = 0.006075\text{m}^4$

$$K_{C4} = \frac{4EI}{L} = \frac{4 \times 26400000 \times 0.006075}{3} = 427680\text{kNm}$$

The simplified moment distribution method is performed as follows. Basically, it is a span-by-span distribution. The sign convention is taken as negative for anti-clockwise moment acting on a beam and positive for clockwise moment. During "carry over" involving moment carried over from one end to the other, the moment direction does not change and therefore no sign change occurs. Taking span B2 dead load as example, the following steps are taken:

(i) The fixed end moments (FEM) are first carried over to the other ends and summed. So for the left end, it is $-49.945 \times 0.167/2 - 49.945 = -54.116$. Similarly for the right end, it is $49.945 \times 0.162/2 + 49.945 = 53.984$.

(ii) The left end moment is then distributed to the column and B1 span by $-(-54.116 \times 0.658) = 35.599$ and $-(-54.116 \times 0.18) = 9.763$ respectively. The moment at B2 left end is the sum of these two.

(iii) The moment of 9.763 at the right end of span B1 is then carried over to the left end by $9.763/2$. It further distributes to the column C1 by $9.763/2 \times 0.755 = 3.684$.

(iv) The process is similar for the right side FEM of B2 to B3;

(v) For B1 that is the left end span, the calculation process stops when the left end FEM distributes on the column junction. For the right side FEM, it will be distributed and carried over through the right two spans. The process is similar for the span B3;

(vi) The final dead load moment at an end will be the sum of all spans;

(vii) The maximum final live load moment at the left end of the abeam is the sum of the moment due to live load on its span and all negative moments contributed by other spans while the minimum live load moment is the sum of the moment due to live load on its span and all positive moments contributed by other spans. The moment signs are reversed for the right end of a beam;

(viii) The dead load shears at the ends of each span are the sum of free shears (calculated by assuming the span is simply supported) plus the elastic shear, which is the sum of the end moments divided by the span length to achieve equilibrium. For the live load shear, the sum of moments is calculated by adding the maximum moment on one end and minimum moment at the other end for the maximum shear.

Member		**C1**	**B1**		**C2**	**B2**		**C3**	**B4**		**C4**
Section		850x450	400x600		400x600	400x600		400x600	400x600		850x450
Span (L)			5.2		3	5.8		3	6.3		
EI			0.0072			0.0072			0.0072		
4EI/L		427680	139015		506880	124634		506880	114743		427680
DF		0.7547	0.2453	0.1804	0.6578	0.1618	0.1670	0.6792	0.1538	0.2115	0.7885
DL	FEM	-	-54.15	57.245	-	-49.945	49.945	-	-69.727	61.665	-
	CO	-	-5.164	6.642	-	-4.171	4.039	-	-6.522	5.361	-
	Sum	-	-59.314	63.887	-	-54.116	53.984	-	-76.249	67.026	-
	Span1	-	-44.764	52.361	-42.027	-10.334	-4.304	3.509	0.794	0.313	-
	Span2	-	3.684	9.763	35.599	-45.362	44.968	-36.668	-8.301	-3.272	-
	Span3	-	-0.433	-1.149	-4.189	5.337	12.735	51.791	-64.525	52.847	-
	MD	-	-41.513	60.975	-10.616	-50.359	53.399	18.632	-72.031	49.888	-

LL	FEM	-	-19.197	19.197	-	-23.128	23.128	-	-32.684	29.669	-
	CO	-	-1.732	2.355	-	-1.931	1.870	-	-3.138	2.513	-
	Sum	-	-20.929	21.552	-	-25.059	24.998	-	-35.822	32.182	-
	Span1	-	-15.795	17.663	-14.177	-3.486	-1.452	1.184	0.268	0.106	-
	Span2	-	1.706	4.521	16.485	-21.006	20.823	-16.980	-3.844	-1.515	-
	Span3	-	-0.204	-0.540	-1.968	2.508	5.983	24.331	-30.314	25.374	-
	Max Mt	-	-15.998	22.184	-	-24.492	26.806	-	-34.158	25.480	-
	Min Mt	-	-14.089	19.584	-	-18.498	19.372	-	-30.046	23.859	-
Max D+L M		-	-57.315	-57.511	83.160	-	-74.851	80.205	-	-106.189	-
Min D+L M		-	-55.609	-55.602	8.967	-	-68.857	72.770	-	-102.078	-
DL	Fr S	-	55.027	61.951	-	51.668	51.668	-	62.901	56.953	-
	El S	-	-3.743	3.743	-	-0.524	0.524	-	3.515	-3.515	-
	V_D	-	51.284	65.694	-	51.144	52.192	-	66.416	53.438	-
LL	Fr S	-	19.707	19.707	-	23.925	23.925	-	27.022	25.490	-
	El S	-	-0.690	1.557	-	0.883	1.432	-	1.635	-0.725	-
	V_L	-	19.017	21.264	-	24.808	25.357	-	28.657	24.765	-
D+L		-	70.302	86.958	-	75.952	77.550	-	95.073	78.203	-

There is also a practice of ignoring the stiffness of the interior columns in the distribution so that the moment in distribution table is simplified as follows. As such, the support moments on both sides of an interior column are equal.

Member		**C1**	**B1**		**B2**		**B3**		**C4**
Section		-	400x600		400x600		400x600		-
Span (L)		-	5.2		5.8		6.3		-
EI		-	0.0072		0.0072		0.0072		-
4EI/L		0.0162	0.005538462		0.004965517		0.004571429		0.0162
DF		0.7547	0.2453	0.5273	0.4727	0.5207	0.4793	0.2215	0.7885
DL	FEM	-	-54.15	57.245	-49.945	49.945	-69.727	61.665	-
	CO	-	-15.092	6.642	-13.002	11.805	-6.522	16.711	-
	Sum	-	-69.242	63.887	-62.947	61.750	-76.249	78.376	-
	Span1	-	-52.256	30.201	-30.201	-7.238	7.238	2.854	-
	Span2	-	12.524	33.190	-33.190	29.599	-29.599	-11.669	-
	Span3	-	-3.949	-10.466	10.466	39.700	-39.700	61.797	-
	MD	-	-43.681	52.925	-52.925	62.061	-62.061	52.981	-
LL	FEM	-	-19.197	19.197	-23.128	23.128	-32.684	29.669	-
	CO	-	-5.061	2.355	-6.021	5.467	-3.138	7.833	-
	Sum	-	-24.258	21.552	-29.149	28.595	-35.822	37.502	-
	Span1	-	-18.307	10.188	-10.188	-2.442	2.442	0.963	-
	Span2	-	5.800	15.369	-15.369	13.707	-13.707	-5.404	-
	Span3	-	-1.855	-4.917	4.917	18.651	-18.651	29.569	-
	Max Mt	-	-20.163	25.557	-25.557	32.358	-32.358	30.532	-
	Min Mt	-	-12.508	5.271	-10.452	11.265	-16.209	24.166	-
Max D+L M		-	-61.381	-63.844	78.482	-78.482	94.419	-94.419	-
Min D+L M		-	-54.353	-56.189	58.196	-63.377	73.326	-78.270	-
DL	Fr S	-	55.027	61.951	51.668	51.668	62.901	56.953	-
	El S	-	-1.778	1.778	-1.575	1.575	1.441	-1.441	-
	V_D	-	53.249	63.729	50.093	53.243	64.342	55.512	-
	Fr S	-	19.707	19.707	23.925	23.925	27.022	25.490	-

LL	El S		2.864	2.510	2.464	3.777	-1.300	2.273	-
	V_L	-	22.571	22.217	26.389	27.702	25.722	27.763	-
D+L		-	75.820	85.945	76.482	80.945	90.064	83.275	-

It can be seen that the results of the two analyses do not differ significantly in this example.

C.2 Frame Analysis under Lateral Load

The following 3 bay 3 storeys frame under lateral loads is to be analyzed under the assumption that the beams are infinite stiff. In the frame, the interior column I values are 1.5 times that of the exterior columns. With S_s as the cumulated floor shear, the shear on the interior column is $S_s \frac{I_c}{2I_c+2\times1.5I_c} = 0.2S_s$ and on the exterior column, it is $S_s \frac{1.5I_c}{2I_c+2\times1.5I_c} = 0.3S_s$, respectively. The beams are of the same section so that their stiffnesses are inversely proportional to their lengths. The frame is analyzed as follows with the applied floor shear as shown in Fig. (**C.4**).

The procedures of the analysis are described as follows:

(i) Below in each of the floors, the cumulated floor shear is distributed onto the columns in accordance with their shear stiffnesses in terms of I/H^3, where H is the height of the column;

(ii) The moment of each column is then calculated by $S \times H/2$, where S is the column shear. This is under the assumption that the column heads or ends are perfectly restrained from rotation which implies the floor is infinitely stiff as compared with the columns. For example, at the second storey where the cumulated shear is 60 + 30 = 90kN, the shear distributed onto the exterior columns is $90 \times 0.2 = 18$kN and $90 \times 0.3 = 27$kN for the interior columns.

(iii) The sum of column moment at a joint will be distributed to the adjoining beam in accordance with the beam's stiffness (4*EI/L*, where *L* is the beam span length) to achieve equilibrium. For example, at the 3rd joint (counting from the left) at the storey, where the sum of the column moment (upper and lower columns) is $36 + 54 = 90$kNm, the moment distributed to the left beam is $90 \times 1/4(1/4 + 1/5) = 50$kNm and to the right is $90 \times 1/5(1/4 + 1/5) = 40$kNm.

(iv) The beam shear is calculated by the sum of the end moments divided by its span. For example, for the left beam at the second storey where the end moments are 60 and 45, the beam shear is $(60 + 45)/4 = 26.25\text{kN}$.

(v) The column axial loads are simply worked out by the summation of the axial loads from above and the net beam shears.

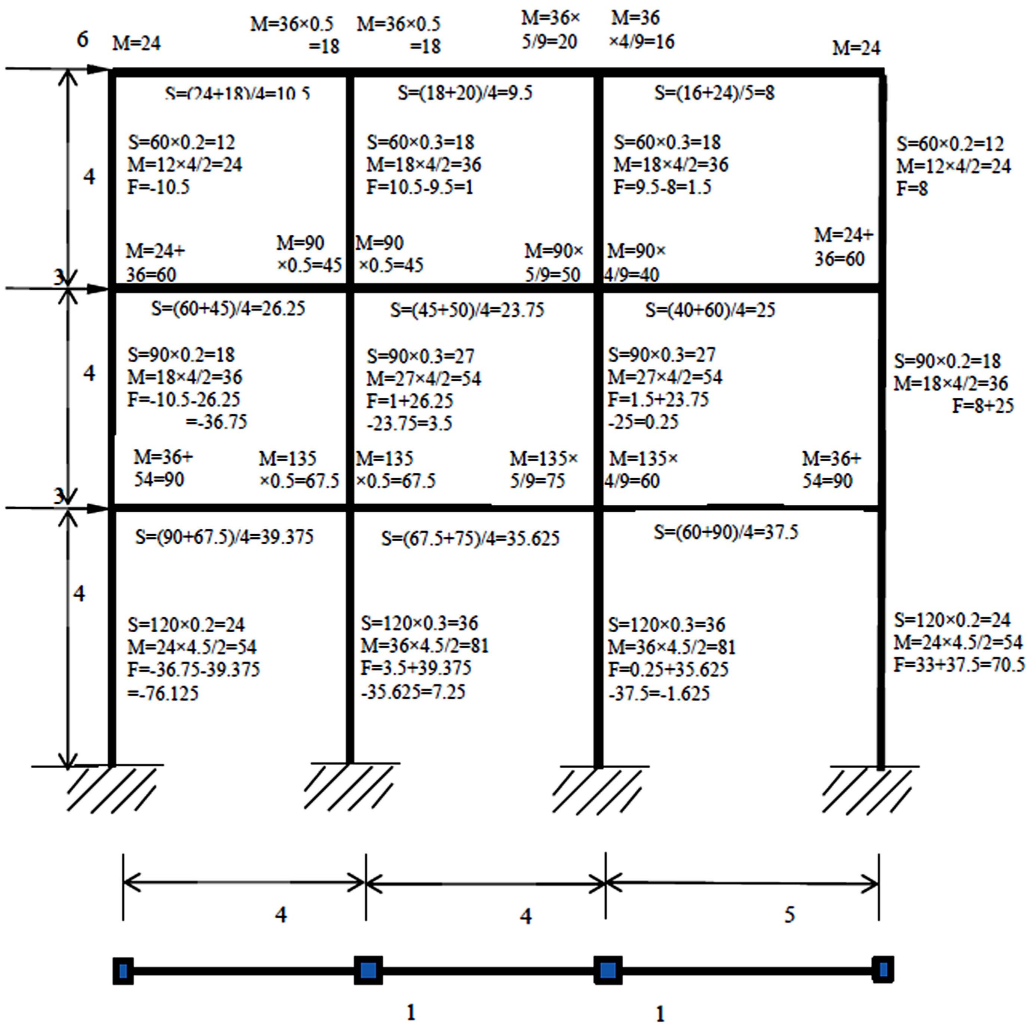


Fig. (C.4). Wind Bent Analysis.

BIBLIOGRAPHY

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APPENDIX D

Derivation of the Stiffness Matrix of a Strut under Axial Load

The shape function of a strut of length L is:

$$v = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad \textbf{(D.1)}$$

$$\theta_z = \frac{dv}{dx} = a_1 + 2a_2 x + 3a_3 x^2 \quad \textbf{(D.2)}$$

Where v is the lateral displacement at x from end 1; $a_0, a_1, a_2, and\ a_3$ are constants depending on the values of v at both ends.

When $x = 0, v = v_1;\ \frac{\partial v}{\partial x} = \theta_{z1};\ x = L, v = v_2,\ \frac{\partial v}{\partial x} = \theta_{z2}$

Solving $a_0 = v_1; a_1 = \theta_{z1};$

Substituting and solving:

$$a_2 = \frac{3v_2 - 3v_1 - 2\theta_{z1}L - \theta_{z2}L}{L^2};\ a_3 = \frac{2v_1 - 2v_2 + 2\theta_{z1}L + \theta_{z2}L}{L^3}$$

$$\therefore v = v_1 + \theta_{z1}x + \frac{3v_2 - 3v_1 - 2\theta_{z1}L - \theta_{z2}L}{L^2}x^2 + \frac{2v_1 - 2v_2 + 2\theta_{z1}L + \theta_{z2}L}{L^3}x^3 \quad \textbf{(D.3)}$$

Re-arranging

$$v = \left(1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}\right)v_1 + \left(x - 2\frac{x^2}{L} + \frac{x^3}{L^2}\right)\theta_{z1} + \left(3\frac{x^2}{L^2} - 2\frac{x^3}{L^3}\right)v_2 + \left(\frac{-x^2}{L} + \frac{x^3}{L^3}\right)\theta_{z2} \quad \textbf{(D.4)}$$

If the strut is under an axial load P, the energy functional can be written as:

$$\Pi = \frac{1}{2}\int_0^L EI_z\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx - S_{y1}v_1 - M_{z1}\theta_{z1} - S_{y2}v_2 - M_{z2}\theta_{z2} - \frac{1}{2}P\delta\ , \quad \textbf{(D.5)}$$

Where δ is the shortening of the strut under the axial P so that the work done is $\frac{1}{2}P\delta$.

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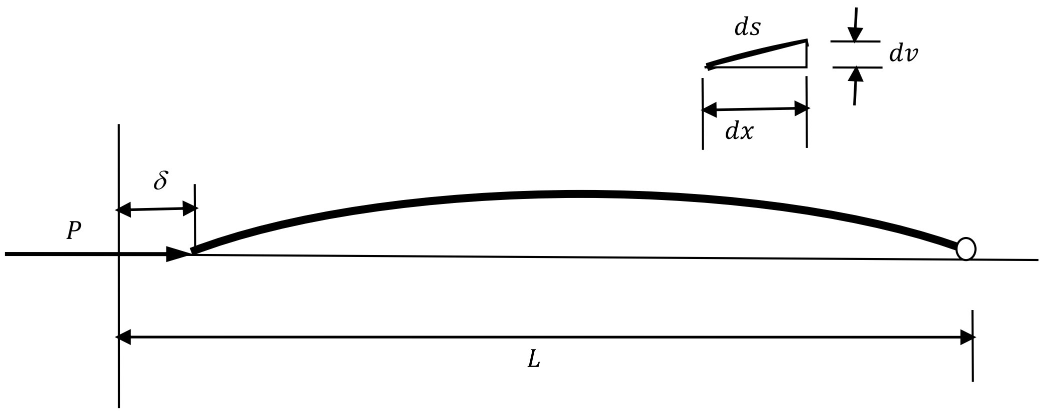

Fig. (C.1). Shortening of strut under effect of axial load.

δ is determined as follows:

$$ds = \sqrt{dx^2 + dv^2} = dx\sqrt{1 + \left(\frac{dv}{dx}\right)^2}$$

$$L = \int_0^L ds = \int_\delta^L dx\sqrt{1 + \left(\frac{dv}{dx}\right)^2} = \int_0^L dx\sqrt{1 + \left(\frac{dv}{dx}\right)^2} - \int_0^\delta dx\sqrt{1 + \left(\frac{dv}{dx}\right)^2}$$

$\approx \int_0^L \left(1 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2\right) dx - \int_\delta^L \left(1 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2\right) dx$ (binomial expansion ignoring orders higher than $\left(\frac{dv}{dx}\right)^2$ which are very small).

$$L = \int_0^L [1]dx + \int_0^L \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx - \int_0^\delta [1]dx - \int_0^\delta \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx$$

$$L = L + \int_0^L \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx - \delta - \int_0^\delta \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx$$

$$\delta = \int_0^L \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx - \int_0^\delta \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx \qquad \textbf{(D.6)}$$

δ is taken as $\int_0^L \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx$ as $\int_0^\delta \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx$ is small.

$$\delta = \int_0^L \frac{1}{2}\left(\frac{dv}{dx}\right)^2 dx \qquad \textbf{(D.7)}$$

Substituting into (D.5), the energy functional is:

$$\Pi = \frac{1}{2}\int_0^L EI_z\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx - S_{y1}v_1 - M_{z1}\theta_{z1} - S_{y2}v_2 - M_{z2}\theta_{z2} - \frac{P}{2}\int_0^L\left(\frac{dv}{dx}\right)^2 dx \quad \textbf{(D.8)}$$

$$\frac{1}{2}\int_0^L EI_z\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx = \frac{2EI_z}{L^3}\left[L^2\left({\theta_{z1}}^2 + \theta_{z1}\theta_{z2} + {\theta_{z2}}^2\right) + 3L(v_1 - v_2)(\theta_{z1} + \theta_{z2}) + 3(v_1 - v_2)^2\right] \qquad \textbf{(D.9)}$$

and

$$\frac{P}{2}\int_0^L\left(\frac{dv}{dx}\right)^2 dx = \frac{P}{30L}\left[2L^2\left({\theta_{z1}}^2 + {\theta_{z2}}^2\right) + 18\left({v_1}^2 + {v_2}^2\right) + 3(v_1 - v_2)(\theta_{z1} + \theta_{z2}) - L^2\theta_{z1}\theta_{z2} - 36v_1v_2\right] \qquad \textbf{(D.10)}$$

Substituting (D.9) and (D.10) into (D.8). For Π to be minimum with respect to $v_1, v_2, \theta_{z1}, \theta_{z2}$, we list:

$$\frac{\partial \Pi}{\partial v_1} = 0$$

$$\Rightarrow \frac{12EI_z}{L^3}v_1 + \frac{6EI_z}{L^2}\theta_{z1} - \frac{12EI_z}{L^3}v_2 + \frac{6EI_z}{L^2}\theta_{z2} - P\left(\frac{6}{5L}v_1 + \frac{1}{10}\theta_{z1} - \frac{6}{5L}v_2 + \frac{1}{10}\theta_{z2}\right) = S_{y1} \qquad \textbf{(D.12)}$$

$$\frac{\partial \Pi}{\partial \theta_{z1}} = 0$$

$$\Rightarrow \frac{6EI_z}{L^2}v_1 + \frac{4EI_z}{L}\theta_{z1} - \frac{6EI_z}{L^2}v_1 + \frac{2EI_z}{L}\theta_{z2} - P\left(\frac{1}{10}v_1 + \frac{2L}{15}\theta_{z1} - \frac{1}{10}v_2 - \frac{L}{30}\theta_{z2}\right) = M_{x1} \qquad \textbf{(D.12)}$$

$$\frac{\partial \Pi}{\partial v_2} = 0$$

$$\Rightarrow \frac{-12EI_z}{L^3}v_1 - \frac{6EI_z}{L^2}\theta_{z1} + \frac{12EI_z}{L^3}v_2 - \frac{6EI_z}{L^2}\theta_{z2} - P\left(\frac{-6}{5L}v_1 - \frac{1}{10}\theta_{z1} + \frac{6}{5L}v_2 - \frac{1}{10}\theta_{z2}\right) = S_{y1} \qquad \textbf{(D.13)}$$

$$\frac{\partial \Pi}{\partial \theta_{z2}} = 0$$

$$\Longrightarrow \frac{6EI_z}{L^2} v_1 + \frac{2EI_z}{L} \theta_{z1} - \frac{6EI_z}{L^2} v_1 + \frac{2EI_z}{L} \theta_{z2} - P\left(\frac{1}{10} v_1 - \frac{L}{30} \theta_{z1} - \frac{1}{10} v_2 + \frac{2L}{15} \theta_{z2}\right) = M_{x1} \qquad \textbf{(D.14)}$$

With the inclusion of the axial load and axial deformation $(u_1 - u_2)\frac{EA}{L} = F_x$, we can write in matrix form as $(k_e + Pk_g)\Delta = F$ **(D.15)**

Where,

$$k_e = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI_z/L^3 & 6EI_z/L^2 & 0 & -12EI_z/L^3 & 6EI_z/L^2 \\ 0 & 6EI_z/L^2 & 4EI_z/L & 0 & -6EI_z/L^2 & 2EI_z/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI_z/L^3 & -6EI_z/L^2 & 0 & 12EI_z/L^3 & -6EI_z/L^2 \\ 0 & 6EI_z/L^2 & 2EI_z/L & 0 & -6EI_z/L^2 & 4EI_z/L \end{bmatrix}$$

$$k_g = \frac{-1}{30L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{bmatrix}$$

k_e is the elastic stiffness matrix we are familiar with while k_g is the geometric matrix used to modify the stiffness of the strut under axial load. Basically the stiffness of the strut represented by $(k_e + Pk_g)$ will be reduced when P is positive (in compression) and increased verse versa.

k_e and k_g can be similarly expanded for a strut in 3-dimensions with 6 degrees of freedom. They can also be similarly transformed when the strut is at certain orientations to the X, Y and Z axes. The 3-dimensional versions of k_e and k_g are listed as follows:

$$k_e =$$

$$\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\
\frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix}$$

$$k_g = \frac{-1}{30L}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 36 & 0 & 0 & 0 & 3L & 0 & -36 & 0 & 0 & 0 & 3L \\
0 & 0 & 36 & 0 & -3L & 0 & 0 & 0 & -36 & 0 & -3L & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -3L & 0 & 4L^2 & 0 & 0 & 0 & 3L & 0 & -L^2 & 0 \\
0 & 3L & 0 & 0 & 0 & 4L^2 & 0 & -3L & 0 & 0 & 0 & -L^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -36 & 0 & 0 & 0 & -3L & 0 & 36 & 0 & 0 & 0 & -3L \\
0 & 0 & -36 & 0 & 3L & 0 & 0 & 0 & 36 & 0 & 3L & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -3L & 0 & -L^2 & 0 & 0 & 0 & 3L & 0 & 4L^2 & 0 \\
0 & 3L & 0 & 0 & 0 & -L^2 & 0 & -3L & 0 & 0 & 0 & 4L^2
\end{bmatrix}$$

$$\Delta = [u_1 \quad v_1 \quad w_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad u_2 \quad v_2 \quad w_2 \quad \theta_{x2} \quad \theta_{y2} \quad \theta_{z2}]^T$$

$$F = [P_1 \quad S_{x1} \quad S_{y1} \quad M_{x1} \quad M_{y1} \quad M_{z1} \quad P_2 \quad S_{x2} \quad S_{y2} \quad M_{x2} \quad M_{y2} \quad M_{z2}]^T$$

APPENDIX E

Structural Design against Fire Resistance – Fire Curves Data and Worked Examples for Structural Member Design against Fire

E.1 Fire Curves

Equations for the fire curves adopted by EN1991-1-2

(i) Standard time-temperature curve: $\theta_g = 20 + 345\log_{10}(8t + 1)$;

(ii) External fire curve: $\theta_g = 660(1 - 0.687e^{-0.32t} - 0.313e^{-3.8t}) + 20$;

(iii) Hydrocarbon curve: $\theta_g = 1080(1 - 0.325e^{-0.167t} - 0.675e^{-2.5t}) + 20$

The HCM Tunnel Curve is $\theta_g = 1280(1 - 0.325e^{-0.167t} - 0.675e^{-2.5t}) + 20$

Table E.1. The RWS Tunnel Curve.

Time (min)	0	3	5	10	30	60	90	120	180	240
Temperature (°C)	20	890	1140	1200	1300	1350	1300	1200	1200	1200

The RABT-ZTV (Train)

Time (min)	0	5	60	170
Temperature (°C)	15	1200	1200	15

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The RABT-ZTV (Car)

Time (min)	0	5	30	140
Temperature (°C)	15	1200	1200	15

ASTM-E117

Time (min)	0	5	10	30	60	120	240
Temperature (°C)	0	538	704	840	927	1010	1093

E.2 Worked Example – Fire Resistance of a Reinforced Concrete Slab to Eurocode En1992-1-2

E.2.1 Check against Tabulated Data

Consider a flat slab of thickness 180mm thick. The concrete mix is C30/37 with reinforcement H12 – 200 ($A_{s,prov} = 565$mm^2/m), axis-distance $= 35$mm. The design rating is R120.

Design data : $h = 200$mm; $a = 35$mm; $f_{ck} = 30$N/mm^2;

$$f_{yk} = 500\text{N/mm}^2; A_{s1} = 565\text{mm}^2/\text{m}.$$

$$d = h - a = 200 - 35 = 165\text{mm}$$

The minimum thickness of the slab and the axis distance a satisfy the tabulated data listed in Table **5.9** of the Eurocode EN1992-1-2 in terms of minimum slab thickness and axis distance.

E2.2 500°C isotherm method by Annex B of Eurocode EN1992-1-2

The design moment of the section is made up of that due to characteristic dead load and live load respectively of 15kNm and 10kNm, which is $M_{E,d} = 1.35 \times 15 + 1.5 \times 10 = 35.25$kNm/m.

For the balance of the section, the concrete compressive depth λx is worked out by

$b\lambda x 0.5667 f_{ck} = 0.87 f_{yk} A_{s1}$

Substituting and solving, λx =14.457mm. $\therefore z = d - \lambda x/2 = 157.77$mm

So the moment of resistance of the section is:

$M_{R,d} = 0.87 f_{yk} A_{s1} z = 38.776$kNm/m > $M_{E,d} = 35.25$kNm

Fig. (**E.1**) of the Eurocode EN1992-1-2 4.2, the steel reinforcement at $a = 35$mm and R120 is 500°C.

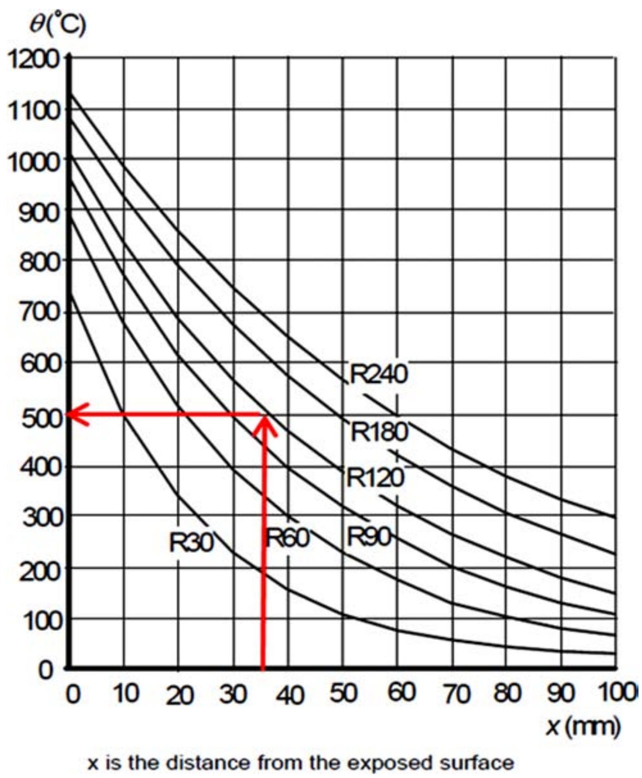

Fig. (E.1). Extraction of Fig. (**A.2**) from EN1992-1-2.

The reduced strength of the bars, f_{sk} is in accordance with Table **3.2a** (or Fig. (**4.2a**) of the Eurocode for Class N (hot rolled) reinforcement.

Corner bar: Temperature: 500°C, $k_s(\theta) = 0.78$, $f_{sk} = 0.78 \times 500 = 390$N/mm^2.

So the reduced section (also with reduced strength of the reinforcement bars) is:

$b\lambda x 0.5667 f_{ck} = 0.87(f_{sk}A_s)$

$\Longrightarrow 1000\lambda x 0.5667 \times 30 = 0.87(390 \times 565)$

$\Longrightarrow \lambda x = 11.28$mm

$z = d - \lambda x/2 = 159.36$mm < $0.95 \times 165 = 156.75$mm

$M_{R,d,fi} = 0.87 f_{sk} A_{s1} z = 0.87 \times 390 \times 565 \times 156.75 \times 10^{-6} = 30.05$kNm/m

The reduction factor due to fire is $\eta_{fi} = \frac{G_k + \psi_{fi} Q_{k,1}}{\gamma_G G_k + \gamma_{Q1} Q_{k,1}} = \frac{15+0.7\times10}{1.35\times15+1.5\times10} = 0.624$

So, the $M_{E,d,fi} = 0.624 \times 35.25 = 21.996$kNm < $M_{R,d,fi} = 30.05$kNm. OK.

E.3 Worked Example – Fire Resistance Of A Reinforced Concrete Beam To Eurocode En1992-1-2

E.3.1 Check against Tabulated Data

Consider a simply supported reinforced concrete beam of size 600(h) x 300(b), singly reinforced. The concrete mix is C30/37 with reinforcement 3H25 ($A_{s,prov} = 1473$mm^2). The design fire rating is R120. The beam is exposed to fire on 3 sides.

Design data: $b = 300$mm;$h = 600$mm$a = 55$mm;$f_{ck} = 30$N/mm^2;

$f_{yk} = 500$N/mm^2;$A_{s1} = 1473$mm^2.

$$d = h - a = 600 - 55 = 545\text{mm}$$

The minimum width of the beam and the axis distance a satisfy the tabulated data listed in Table **5.5** of the Eurocode EN1992-1-2 in terms of minimum beam width and axis distance.

E.3.2 500^{o}C Isotherm Method by Annex B of Eurocode EN1992-1-2

The design moment of the section is made up of that due to characteristic dead load and live load of 150kNm and 50kNm, respectively, which is $M_{E,d} = 1.35 \times 150 + 1.5 \times 50 = 277.5$kNm.

The section complies with the tabulated data listed in Table **5.5** for R 120.

For balance of section, the concrete compressive depth λx is worked out by

$b\lambda x 0.5667 f_{ck} = 0.87 f_{yk} A_{s1}$

Substituting and solving, λx =125.63mm. $\therefore z = d - \lambda x/2 = 482.18$mm

So, the moment of resistance of the section is,

$M_{R,d} = 0.87 f_{yk} A_{s1} z = 308.96$kNm > $M_{E,d} = 277.5$kNm.

By the simplified calculation method in 4.2 of the Eurocode EN1992-1-2 4.2 "Reduced cross section and Reduced strength" approach, reference is made to Fig. (**A.8**) of Annex A of the Eurocode for the temperature profiles after 120mm minutes by the standard fire, the cross section is reduced to $b_{ci} = 300 - 32 \times 2 = 236$mm and the temperatures of the 3 number of steel bars are: corner bar : 600^{o}C and mid-bar 400^{o}C as estimated from Fig. (**E.2**). For sections other than that given in the Eurocode, the temperature profiles can be estimated based on the sections given or by first principles based on software.

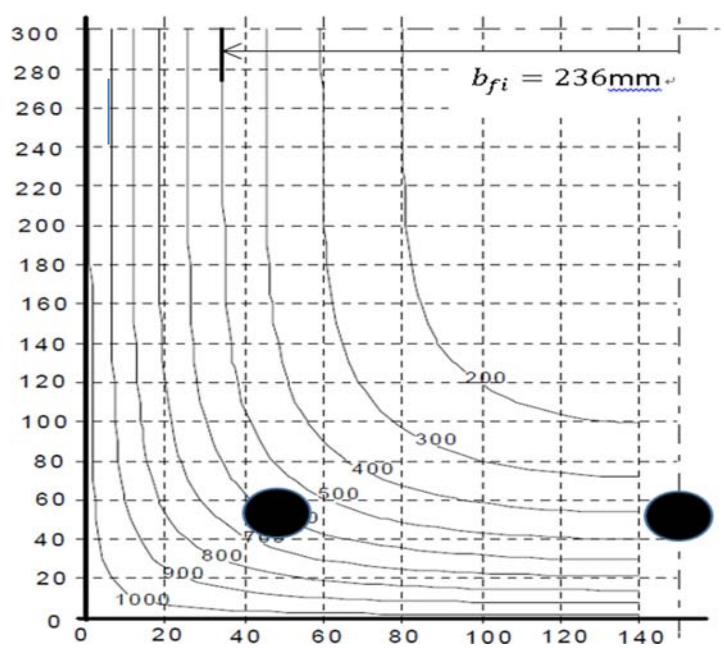

Fig. (E.2). Extraction of Figure A.8 from EN1992-1-2.

The reduced strength of the bars, $f_{sk,corner}$ and $f_{sk,mid}$ is in accordance with Table **3.2a** (or Fig. (**4.2a**) of the Eurocode for Class N (hot rolled) reinforcement.

Corner bar: Temperature: 600ºC, $k_s(\theta) = 0.47$ and mid-bar 400ºC, $k_s(\theta) = 1.0$.

So, the reduced section (also with reduced strength of the reinforcement bars) is:

$$b_{fi}\lambda x 0.5667 f_{ck} = 0.87\left(2 f_{sk,corner} A_{s,corner} + f_{sk,mid} A_{s,mid}\right)$$

$$\Longrightarrow 236\lambda x 0.5667 \times 30 = 0.87(2 \times 490.87 \times 0.47 \times 500 + 490.87 \times 1 \times 500)$$

$$\Longrightarrow 236\lambda x 0.5667 \times 30 = 414245.19$$

$$\Longrightarrow \lambda x = 103.25\text{mm}$$

$$z = d - \lambda x/2 = 493.38\text{mm}$$

$$M_{R,d,fi} = 0.87 f_{yk} A_{s1} z = 414245.19 \times 493.38 \times 10^{-6} = 204.38\text{kNm}$$

The reduction factor due to fire is $\eta_{fi} = \frac{G_k + \psi_{fi} Q_{k,1}}{\gamma_G G_k + \gamma_{Q1} Q_{k,1}} = \frac{150 + 0.7 \times 50}{1.35 \times 150 + 1.5 \times 50} = 0.667$.

So, the $M_{E,d,fi} = 0.667 \times 277.5 = 185$kNm < $M_{R,d,fi} = 204.38$kNm. OK

E.3.3 Zone Method by Annex B of Eurocode EN1992-1-2

The beam is divided into 5 zones on half width with mean temperatures estimated as below. The zoning at the bottom of the beam is not necessary as the concrete strength is ignored due to tension. The corresponding $k_c(\theta)$ is read from Fig. (**4.1**) of the Eurocode EN1992-1-2 (which has been extracted in Fig. (**5.17**) in Section 5):

The mean reduction coefficient for the section is (By equation B.11 of BSEN1992-1-2)

$$k_{c,m} = \frac{(1 - 0.2/n)}{n} \sum k_c(\theta_i) = \frac{(1 - 0.2/5)}{5}(0.14 + 0.75 + 0.92 + 0.96 + 0.96) = 0.716 .$$

So, the reduced section width is $0.716 \times 300 = 214.8$mm < 236mm obtained in the 500ºC isotherm method.

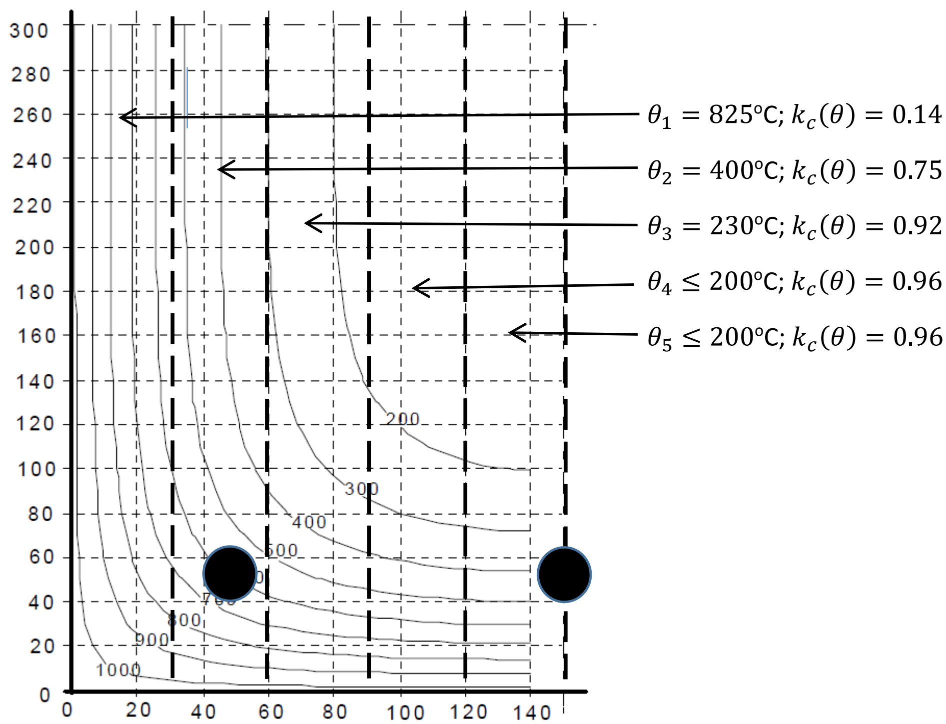

Fig. (E.3). Zoning of the Beam in Fig. (**E.2**).

The bending strength of the section is similarly worked as follows under $b_{fi} =$ 214.8mm

$$b_{fi}\lambda x 0.5667 f_{ck} = 0.87\left(2f_{sk}A_{s,corner} + f_{sk,}A_{s,mid}\right)$$

$$\Longrightarrow 214.8\lambda x 0.5667 \times 30 = 414245.19$$

$$\Longrightarrow \lambda x = 113.44\text{mm}$$

$$z = d - \lambda x/2 = 488.28\text{mm}$$

$$M_{R,d,fi} = 0.87 f_{yk} A_{s1} z = 414245.19 \times 488.28 \times 10^{-6} = 202.27\text{kNm} > M_{E,d,fi} = 185\text{kNm, OK.}$$

E.4 Worked Example – Fire Resistance of a Structural Steel Member to Eurocode En1993-1-2

A 200mm thick concrete slab is supported on 3 I-beams 406×178×74 UB of grade S275 as shown in Fig. (**E.4**).

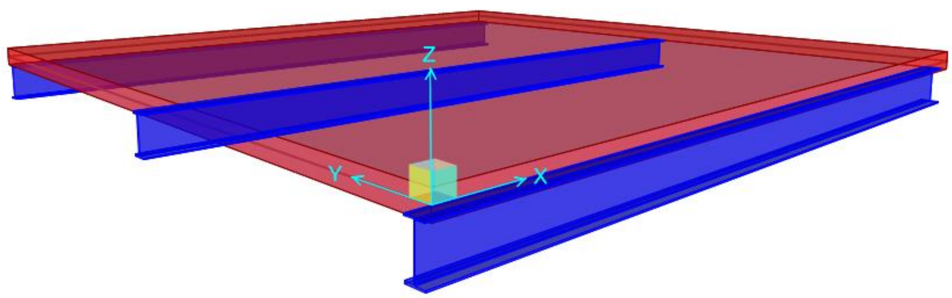

Fig. (E.4). Steel Beam for Fire Resistance Study.

The sectional properties of the structural steel beam are given as follows:

$A = 0.00945\text{m}^2; I_3 = 0.0002731\text{m}^4; I_2 = 0.00001545\text{m}^4;$

$W_{el3} = 0.001323\text{m}^3; W_{el2} = 0.0001721\text{m}^3;$

$W_{pl3} = 0.001501\text{m}^3; W_{pl2} = 0.0002670\text{m}^3;$

$B = 0.1795\text{m}; D = 0.4128\text{m}; T = 0.016\text{m}; t = 0.0095\text{m}$

The beams are simply supported of span = 8m supporting udl due to permanent load and live load.

The following steps are taken to determine the protection required for the beam to achieve R120.

(i) The characteristic loads during fire are taken as:
Permanent Load $G_k = 20\text{kN/m}$; Live load $Q_{k,1} = 10\text{kN/m}$.
The bending moment is:
$M_{Ed} = (1.35 \times 20 + 1.5 \times 10) \times 8^2/8 = 42 \times 8^2/8 = 336\text{kNm}$
The udl during fire is $q_{fi,d,t} = G_k + \psi_{2,1} Q_{k,1} = 20 + 0.6 \times 10 = 26\text{kNm}$

The mid-span bending moment is $M_{fi,d,t} = 26 \times 8^2/8 = 208$kNm.
The end span shear is $V_{fi,d,t} = 26 \times 8/2 = 104$kN.

(ii) To classify the member:
$\varepsilon = 0.85\sqrt{235/f_y} = 0.85\sqrt{235/275} = 0.786$;
$B/2T = 5.61 < 9\varepsilon = 7.07; \frac{d}{t} = 37.4 < 72\varepsilon = 56.59$
By Table **5.2** of Eurocode EN 1993-1-1, it is a Class 1 section

(iii) The resistance of the section at ambient temperature is
$M_{Rd} = M_{pl,Rd} = W_{pl,y}f_y/\gamma_{M0} = 0.001501 \times 275000/1 = 412.78$kNm
$V_{Rd} = V_{pl,Rd} = A_v f_y/\sqrt{3}/\gamma_{M0} = 0.4128 \times 0.0095 \times 275000/\sqrt{3}/1 = 622.64$kN

(iv) To determine the degree of utilisation,
$\mu_{0,M} = M_{fi,d,t}/M_{Rd} = 208/412.78 = 0.504$
$\mu_{0,V} = V_{fi,d,t}/V_{Rd} = 104/622.64 = 0.167$.

So, the bending moment controls the design.

(v) The critical temperature is calculated (with bending moment being more critical) as (Eqn 4.22 of the Eurocode EN1993-1-2)
$\theta_{a,cr} = 39.19\ln\left(\frac{1}{0.9674\mu_0{}^{3.833}} - 1\right) + 482 = 39.19\ln\left(\frac{1}{0.9674\times0.504^{3.833}} - 1\right) + 482 = 583.38$ °C.
Though there is a step by step procedure for determining the time to reach the critical temperature by the unprotected beam, the time for a bare steel member should be very short far less than 120 min. So, this part of check can be skipped.
(vi) To find the time taken by the I-beam to reach the critical temperature under coating of a certain thickness of an insulation which is vermiculite cement, the properties of the coating are listed as follows:

Density: $\rho_p = 350$kg/m^3;Specific heat $c_p = 1200$J/kg°K;

Conductivity: $\lambda_p = 0.12$W/m°Kthickness $d_p = 12$mm

Density and specific heat of steel taken as $\rho_a = 7850$kg/m^3 and $c_a = 600$J/kg°K in accordance with 3.4.1.2 of EN1993-1-2

$A_p = 4B + 2D - 2t = 1.5246$m (perimeter of the I-section)

$V = 0.00945$m^2, cross sectional area of the I-section

$$\therefore \frac{A_p}{V} = \frac{1.5246}{0.00945} = 161.33$$

A parameter $\phi = \frac{c_p \rho_p \, d_p}{c_a \rho_a} \frac{A_p}{V} = \frac{1200 \times 350 \times 0.012}{420 \times 7850} \times 161.33 = 0.1726$ is used.

The following equation for determining the temperature of the I-beam with the above protection under a standard fire is (Eqn 4.27 of the Eurocode EN1993-1-2).

$$\Delta\theta_{a,t} = \frac{\lambda_p / \, d_p}{c_a \rho_a} \frac{A_p}{V} \left(\frac{1}{1 + \phi/3} \right) \left(\theta_{g,t} - \theta_{a,t} \right) \Delta t - \left(e^{\phi/10} - 1 \right) \theta_{g,t}$$

where $\theta_{a,t}$ is the temperature of the steel member at time t and $\Delta\theta_{a,t}$ is the change in temperature in the steel in the time step for Δt from t.

The analysis is conducted as per the following assumption:

(1) The steel is being heated in accordance with the Standard Fire Curve of the Eurocode EN1991-1-2

(2) Using a time step of 1 minute;

(3) Starting with an initial temperature of 20^{o}C for both the steel member and the standard fire;

(4) Using the $\theta_{a,t}$ determined in the previous step to calculate $\Delta\theta_{a,t}$ for the next time step.

Parts of the tabulated results for calculation based on 12mm thick vermiculite cement coating care extracted as follows:

t (min)	$\theta_{g,t}$	$\Delta\theta_{a,t}$	$\theta_{a,t}$	t (min)	$\theta_{g,t}$	$\Delta\theta_{a,t}$	$\theta_{a,t}$
0	20	0	20	58	940.27	7.34	567.79
1	349.21	0.67	20.67	59	942.83	7.24	575.03
2	444.50	6.58	27.24	60	945.34	7.15	582.18

3	502.29	8.23	35.47	61	947.81	7.06	589.25
4	543.89	9.16	44.62	62	950.24	6.97	596.22
5	576.41	9.77	54.39	63	952.64	6.88	603.10

Under interpolation, the time for reaching the critical temperature of 583.38^{o}C is 60.17 min attaining R60.

If we aim at R120, the coating has to be increased to a thickness of 26mm, similar parts of the tabulated results are shown as follows:

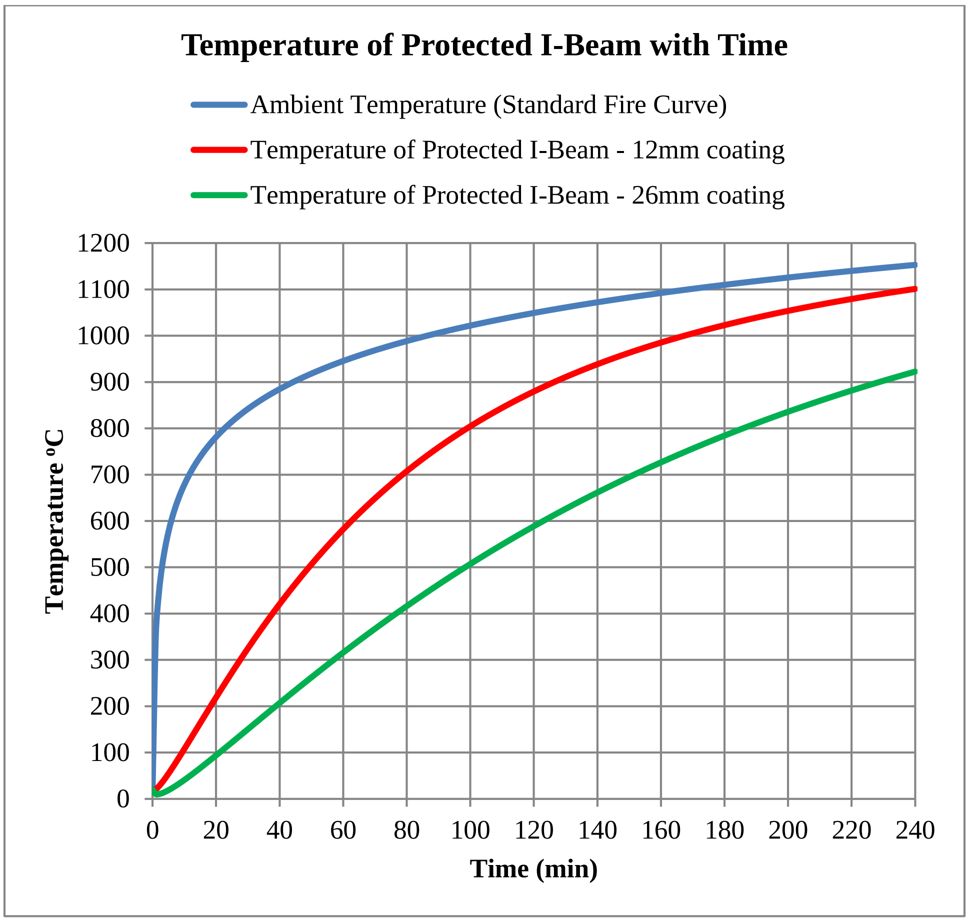

Fig. (E.5). Plots of Temperature of the Protected Beams with Time.

t (min)	$\theta_{g,t}$	$\Delta\theta_{a,t}$	$\theta_{a,t}$	t (min)	$\theta_{g,t}$	$\Delta\theta_{a,t}$	$\theta_{a,t}$
0	20	0	20	118	1046.52	3.91	580.94
1	349.21	-9.77	10.23	119	1047.79	3.89	584.83
2	444.50	0.03	10.26	120	1049.04	3.87	588.69
3	502.29	1.95	12.21	121	1050.28	3.85	592.54
4	543.89	2.90	15.11	122	1051.51	3.82	596.36
5	576.41	3.49	18.60	123	1052.74	3.80	600.17

By similar interpolation, the time to reach the critical temperature is 118.63^{o}C which is close to 120^{o}C or attaining R120.

Plots of the two coating together with the standard fir curve are shown as follows:

By the procedure listed in (vi), the fire rating for load bearing can be determined.

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