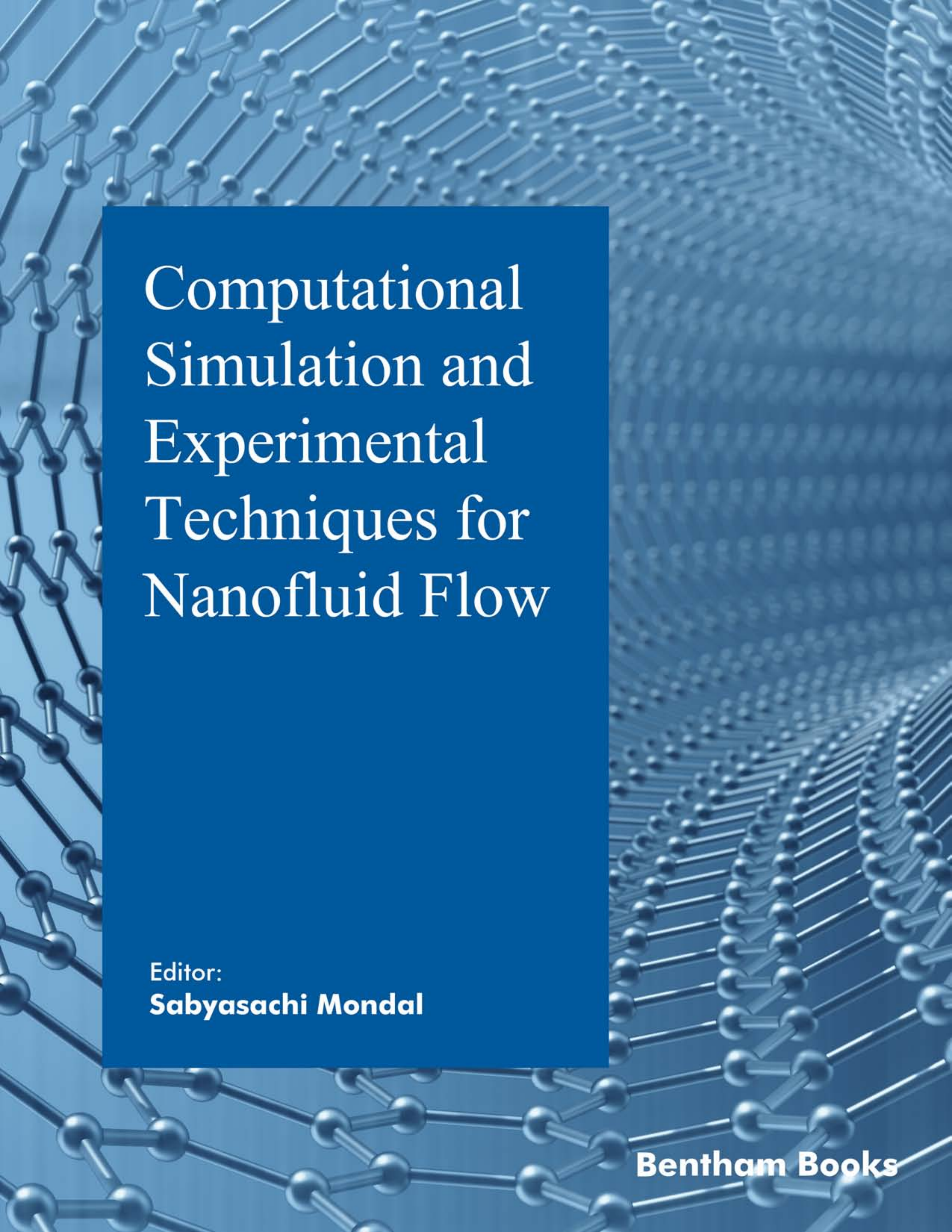

Computational Simulation and Experimental Techniques for Nanofluid Flow

Editor:
Sabyasachi Mondal

Bentham Books

Computational Simulation and Experimental Techniques for Nanofluid Flow

Edited by

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Editor: Sabyasachi Mondal

ISBN (Online): 978-981-5223-70-5

ISBN (Print): 978-981-5223-71-2

ISBN (Paperback): 978-981-5223-72-9

First published in 2024.

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CONTENTS

PREFACE

Advancements in the field of nanofluid flow have revolutionized numerous industries, ranging from energy and biomedical applications to electronics and manufacturing. The intricate behavior and unique properties of nanofluids have captured the imagination of researchers worldwide, sparking an intense exploration of their fundamental characteristics and practical applications. As a result, the need for comprehensive resources that bridge the gap between computational simulation and experimental techniques has become increasingly evident.

This edited volume, titled "Computational Simulation and Experimental Techniques for Nanofluid Flow," represents a collective effort of distinguished experts and researchers who have devoted themselves to unraveling the complexities of nanofluid dynamics. Drawing on their extensive experience and deep understanding of the subject, these contributors have compiled a rich compendium of knowledge that covers a wide spectrum of topics within this burgeoning field. The book is structured to provide a holistic overview of both computational and experimental methodologies, offering readers a balanced perspective on the subject matter. By exploring the synergies and complementarities between these two approaches, the editor and authors seek to foster a deeper appreciation for the challenges and possibilities that arise when investigating nanofluid flow phenomena.

The first section of the book delves into the theoretical foundations of nanofluid dynamics, providing a comprehensive framework to understand the principles underlying the behavior of nanoparticles in fluid environments. Various aspects of non-Newtonian behavior, hybrid nanofluids, and their daily life applications are discussed in the initial chapters, emphasizing the high heat transfer rates and enhanced thermal conductivity offered by hybrid nanomaterials. Furthermore, novel computational techniques such as the overlapping grid multi-domain bivariate spectral simple iteration method (OMD-BSSIM) are employed to solve hybrid nanofluid flow problems, utilizing the Chebyshev spectral collocation approach to handle the challenges of multi-domain problems.

Continuing with numerical investigations, the subsequent chapters explore the development of theoretical models and computational simulations for nanofluid flow phenomena. The third chapter focuses on the development of a theoretical model solved numerically using the Spectral Quasilinearization method. In the fourth chapter, the impinging oblique flow of nanofluids past a stretching sheet is analyzed, employing a cutting-edge Genetic Algorithm technique to understand the thermal characteristics and heat transport behavior over time. The fifth and sixth chapters concentrate on the behavior of Casson nanofluids and magneto-radiative nanofluids over different sheets, employing distinct numerical methods to solve the respective problems. The final two chapters investigate new nanofluid flow models and utilize the Spectral Quasi-Linearization Method (SQLM) to solve the governing equations, briefly discussing the influence of thermophoresis and Brownian motion.

Summarizing the discussions of all the chapters, the first section of the book establishes the theoretical foundations of nanofluid dynamics, providing a comprehensive framework for understanding the behavior of nanoparticles in fluid environments. Moving forward, the second section focuses on computational simulation techniques employed to study nanofluid flow, showcasing state-of-the-art methods such as MATLAB bvp4c, RK Method, cutting-edge Genetic Algorithm, Spectral Quasi-Linearization Method (SQLM), and the overlapping grid multi-domain bivariate spectral simple iteration method (OMD-BSSIM). These discussions shed light on the strengths and limitations of these methods in capturing the intricate behavior of nanofluids. The final section of the book addresses emerging trends and future directions in the field, exploring the potential impact of nanofluid flow on key industries and proposing avenues for further research and innovation.

As the editor, I aim to offer a comprehensive reference that stimulates the imagination, curiosity, and collaborative spirit of researchers, practitioners, and students embarking on their journey into the captivating world of nanofluid flow.

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CHAPTER 1

Cattaneo-Christov Heat Flux based Darcy-Forchheimer Hybrid Nanofluid Flow with Marangoni Convection above a Permeable Disk

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Abstract: The study highlights a rising number of fluids such as nanofluids and hybrid nanofluids encountered in daily life that exhibit non-Newtonian behavior and they are exploited in manufacturing due to their high heat transfer rate becoming more and more important as time goes on. The focus is on hybrid nanomaterials because they increase liquid alloys' and fluids' thermal conductivity. The various investigations on a thermal Marangoni convective flow of aluminium alloy and Boehmite alumina nanoparticles into gasoline oil in base fluid water on a steady Darcy-Forchheimer flow are covered. With the system's exponential heat generation and viscous dissipation, the thermal impact is more pronounced in the presence of Cattaneo-Christov heat flux. To simplify the highly coupled nonlinear governing equations (PDEs) and the boundary conditions (BCs), a suitable similarity conversion is being applied. The outcomes of the conversion equations and their BCs are evaluated by MATLAB bvp4c routine with the shooting techniques. Through the use of graphs and tables, it has been determined how different governing factors affect the velocity, temperature, skin friction, and Nusselt number. A quick comparison between a hybrid nanofluid and a nanofluid is displayed in each graph. We also discussed the system's Bejan numbers and entropy creation. It has been observed that the proportion of heat transmission increases with heat generation but decreases with the Marangoni number. A nonlinear increase in the permeability constant and Brinkman number results in a rise in entropy generation.

Keywords: Catteneo-Christov heat flux, Darcy-Forchheimer flow, Entropy generation, Hybrid nanofluid, Marangoni convection, Nanoflui.

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INTRODUCTION

Nowadays, the study of nanofluid and hybrid nanofluid has become the hottest topic among researchers and scholars.

Several researchers conveyed their opinion regarding hybrid nanofluids with a high heat transmission rate. In order to speed up the rate of heat transmission, Choi *et al*. [1] first presented the idea of a nanofluid. The models of the hybrid nanofluid deal with new exciting possibilities to enhance the heat transfer of fluid and demonstrate the characteristics for future applications in the field of biochemical engineering, biomedical engineering, and in detergents other than non-Newtonian fluids. Recently Nayak *et al*. [2] examined entropy generation in hybrid fluid flow between two parallel plates. After that, many researchers deliberated on hybrid-fluidity experimentally and numerically [3–5]. The idea of a hybrid nanofluid is taken into consideration to aggregate two different nanoparticles to enhance thermal conductivity [6-9]. Suresh *et al*. [10, 11] introduced the concept of hybrid nanofluid and its concept provides a broad area for researchers to provide their worthy efforts. Baghbanzadeh *et al*. [12] also analyzed the accumulation of spherical silica nanotubes. Nine *et al*. [13] explored the Al_2O_3−MWCNTs and the recent study on hybrid nanofluid subjects to consider the thermal properties [14-21]. An inexhaustible and pollution-free solar radiation energy is utilized to fulfill the increasing demand [22]. The conjugate influence of entropy generation and thermal radiation over various geometries was studied by several researchers [23-28]. At present, concentrated solar collectors are used to provide energy for low- and medium-temperature processes, including improved absorption solar refrigeration systems [29], air cooling [30], and water heating [31]. In general, turbulent flows increase the fluid flow and heat transfer [32]. To propose better models for turbulent nanofluid flow, some researchers have compared them [33–35]. Nanofluids can use the current model for heat and mass transfer over different geometries [36–39].

Despite several works on Darcy-Forchheimer law in dissimilar non-Newtonian viscoelastic liquid inspected by countless investigators, for instance, Darcy-Forchheimer, viscous dissipation, gasoline oil, and biometer alumina were investigated by Mohanty *et al*. [40]. Physical characteristics of double chemically reacting hybrid nano-fluid flow over the stretched media dealing with MHD and Darcy Forchheimer law have been examined by Rashidi *et al*. [41]. Mahian *et al*. [42] and Goodarzi *et al*. [43] utilized Darcy Forchheimer's law in a variable tricked surface of the disk and flow across a curved surface. Siddiqui *et al*. [44] probed a rotating and stretched flow of bi-convective nanofluids subject to Darcy

Forchheimer law. The significance of Darcy Forchheimer's law on the dynamics of magnetized nanofluid in a circular form has been examined by Ali *et al*. [45].

In the current investigations, we explored the effect of Darcy-Forchheimer's theory's influence on the porous rotating disk with magnetic field, thermal radiations, and accumulations of Boehmite alumina into gasoline oil. The primary objectives of this work are (a) To analyze the influence of the Marangoni convection for a Darcy-Forchheimer hybrid nanofluid flow across a stretching disk, (b) Heat generation of the system with Catteneo-Christov heat flux and non-linear thermal radiation, (c) The entropy of the system to understand heal losses, and (d) To carry out an assessment of general and hybrid nanofluid on the system and entropy. The effort presented is new as per the literature assessment. Developed partial differential equations (PDEs) demonstrate that the fluid flow is changed into a set of ODEs by applying a resemblance conversion. A numerical procedure namely BVP4C is executed to find results. Attained solutions are evaluated and concluded in tables and graphs.

Terminology

(u, w)	Velocity components
ρ_{nf}	Density of nanofluid
$\left(\rho C_p\right)_{nf}$	Heat capacitance of nanofluid
$\left(\rho C_p\right)_f$	Thermal capacity of the underlying fluid
$\left(\rho C_p\right)_s$	Heat capacity of solid nanoparticles
ρ_s	Density of solid nanoparticles
ρ_f	Fluid's base density
μ_{nf}	Dynamic viscosity of nanofluid

CHAPTER 2

Three-Dimensional Numerical Computational Model for an Unsteady Hybrid Nanofluids Past Stretching MHD Rotating Sheet

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Abstract: This paper presents a numerical approach to solving unsteady hybrid nanofluid flow problems using the overlapping grid multi-domain bivariate spectral simple iteration method (OMD-BSSIM). The method utilizes the Chebyshev spectral collocation approach to approximate the derivatives in the overlapping grid of space and non-overlapping grid of time, which allows for handling the two domains (multi-domain) problem. In this paper, we propose using the overlapping bivariate spectral method in combination with the simple iteration method instead of the commonly used quasi-linearization, relaxation, or local linearization schemes. The governing equations are transformed into a system of nonlinear partial differential equations using similarity transformations. The OMD-BSSIM is applied to investigate the heat transfer rate for MHD, unsteady $GNP - Fe_3O_4/H_2O$, and $TiO_2 - Fe_3O_4/H_2O$ hybrid nanofluids flow, with projection angles ranging from $0°$to$90°$, representing the influence of different magnetic fields. The numerical solution is obtained using OMD-BSSIM implemented in MATLAB. We use R visualization techniques and graphs to analyze the relationship between the skin friction coefficients, local Nusselt number, and Sherwood number of the hybrid nanofluids and some parameters. The results show that the increase in the volume fraction of GNP nanoparticle has a greater effect on the temperature profile than TiO_2 nanoparticle. Additionally, notable positive relationships were observed for the rotation parameter, while the stretching parameter had a negative impact on certain outcome measures.

Keywords: Hybrid nanofluid, Overlapping chebyshev spectral collocation, Rotating surface, Simple iteration method.

INTRODUCTION

In 1961, Sakiadis [1, 2] conducted studies on boundary layer flow for continuously moving surfaces, which included flat and cylindrical surfaces. His research

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introduced a new class of boundary layer problems with solutions that have a wide range of applications. The investigation of boundary layer flow has since become an important area of research in fluid mechanics, with numerous studies exploring the behaviour of boundary layers and their impact on heat transfer. Furthermore, Sakiadis's focus is on both laminar and turbulent flows contributed to the expansion of the field. In 1970, Crane [3] furthered Sakiadis's research by considering a stretching surface instead of a rigid one. Subsequently, multiple authors investigated these characteristics for different types of fluids, each utilizing a unique set of fluid flow properties and working with specific objectives, parameters, and assumptions.

Numerous studies have investigated stretching boundary layer flows [4, 5]. Recently, the use of nanofluids and hybrid nanofluids has gained significant popularity in contemporary industrial applications. The concept of nanofluids was introduced by Choi and Eastman in their pioneering work in 1995 [6]. A nanofluid is composed of a base fluid mixed with a single nanoparticle (NP) [7, 8]. Conversely, a hybrid nanofluid contains one or more additional NPs [9, 10]. Since 1995, these fluids have found a diverse range of industrial and domestic applications. Hybrid nanofluids, for instance, have significant applications in transformer cooling, building heating, and biomedical drug reduction, among others. Moreover, literature has demonstrated that the addition of nanoparticles into a liquid enhances its electrical, thermal, and acoustical conductivity [11]. Nanoparticles are materials characterized by their extremely small size, usually ranging from 1 to 100 *nm* [12].

In recent years, there has been significant interest in the development of materials that exhibit superconductivity at room temperature. This research area has garnered the attention of several scientists, who have been working on this intriguing field. Empirical studies have demonstrated that transition-metal oxides have promising thermoelectric properties. The development of such materials would enable the creation of cooler electronic components and significantly enhance the efficiency of the electricity grid. Through experimental investigations, Ma *et al.* [13] have made attempts to improve the hydrophobicity of graphene (GNP) nanoplatelets by fictionalization, without compromising their good thermal properties. This development has led to the creation of novel hybrid nanofluids that enhance heat transfer. Moreover, it effectively prevents the separation of composite nanoparticles during the boiling process. Ottofuelling *et al.* [14] investigated the colloidal stability of titanium dioxide (TiO_2) nanoparticles in different water environments and identified several factors that affect their aggregation behaviour. Their study demonstrates that the TiO_2 nanoparticle's stability is influenced by ionic strength, and the presence of natural organic matter and divalent ions. Meanwhile, Othman *et al.* [15] have shared an extraordinary finding that water vapor can induce superconductivity in iron arsenic-containing strontium at 25 K. Water appears to cause a kind of chemical or structural change that prompts the transition at a higher temperature.

Hannes Alfvén [16] is credited with formally introducing the field of magnetohydrodynamics (MHD). The MHD defines and studies the dynamics of electrically conducting fluids, mainly focusing on the control of heat transfer. Several authors have conducted numerical investigations on heat transfer properties of the MHD boundary layer flow. For example, Raza *et al.* [17] examined a micro-polar and MHD flow past stretching and shrinking sheets in a porous media with thermal radiation and suction/injection effects. They concluded that the temperature was directly proportional to the thermal radiation parameter, magnetic parameter, and Darcy number. On the other hand, it was inversely proportional to the suction parameter. Furthermore, Khan and Nadeem [18] studied double-stratified rotating Maxwell nanofluid flowing over both linear and exponential stretching sheets. They reported that an increase in the rotation and or relaxation parameters decreased fluid velocity but increased the temperature and related boundary layer thickness.

In this study, we investigated numerically two types of hybrid nanofluids, $GNP - Fe_3O_4/H_2O$, and $TiO_2 - Fe_3O_4/H_2O$ in changing magnetic angles. We utilized the overlapping grid multi-domain bivariate spectral simple iteration method (OMD-BSSIM). The modified method is a combination of two methods, the simple iteration method (SIM) as a linearizing technique and the overlapping multi-domain spectral collocation discretization. The spatial domain is divided into overlapping sub-domains, whereas the temporal domain is divided into non-overlapping sub-domains. The overlapping grid procedure leads to dense matrix equations and was recently proposed by Mkhatshwa *et al.* [19]. Some studies have validated the use of overlapping grid procedures [20, 21] to solve the nonlinear partial differential equations (N-PDEs). The SIM technique was proposed by Motsa *et al.* [22], and it is based on fixed point iteration to develop iterative schemes. The SIM has demonstrated practicality in solving differential equations with applications in fluid mechanics [23].

MATHEMATICAL MODEL

The 3D, MHD, and unsteady flow of an electrically conductive, incompressible, rotating, stratified Fe_3O_4/H_2O hybrid nanofluids flow past a stretching sheet. The flow is defined at $z_3 \geq 0$. Fig. (**1**) shows the coordinate system and the physical

CHAPTER 3

Effects of Activation Energy and Thermo-Convection of Nanofluid Flow

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Abstract: Nonlinear mixed convection in magnetohydrodynamic thermo-convection nanofluid flow is addressed. Flow is induced by a stretched surface. Chemical reaction effects on the activation energy of the species concentration of Arrhenius are considered. The nonlinear governing equations are converted to a dimensionless ordinary system through adequate transformations. The solutions of nonlinear dimensionless expressions are computed by employing the spectral quasilinearization method. A consequence of various parameters involved such as velocity, concentration, and thermal field has been reported graphically. Concentration has the reverse trend for both random motion and thermophoresis variables.

Keywords: Arrhenius energy, Activation energy, SQLM (Spectral Quasilinearization method), Thermos-convection.

INTRODUCTION

Current uses of the borderline fluid stream over a continuous stretching sheet include the production of paper, glass fibre, polymer extrusion, wire and plastic film manufacturing, and many more industries where the fluids' freeze and heat properties are controlled, which grab the eyeballs of the many researchers. From the time it was introduced by Lewig Prandtl in 1904, various studies by different researchers have been carried out [1-5]. Boundary layer fluid flows are commonly used in a variety of engineering applications, including geophysical drifts, groundwater hydrology, petroleum lochs, electronic system refrigeration, porcelain processes, categorization processes, current padding, groundwater smog, and compound catalytic reactors. Various researchers like [6-7] examine the non-Darcy porous medium with micro-convection nanofluids. Various researchers [8-12] discussed classifying the motile microorganism with viscous dissipation and

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activation energy. The effect of activation energy and thermo-convective nanofluid flow is being addressed by many scholars [13-16]. Investigation of the Nanofluid fluid depends on different base fluids that are important at present days for industrial as well as for engineering sectors. Nanofluids are made of nanosized oxide or metal oxides mixed in a proper base fluid like water, or ethylene glycol. These special kinds of fluids are significant to enhance the heat and mass transfer rate. In biomedical studies, the use of nanofluids is also extremely significant.

The chemical reaction is subject to Arrhenius' activation energy with magnetic parameters including heat generation as a new concept. Velocity and micro-convective boundary conditions are considered here. Brownian motion is considered in this study as a parameter having a great influence on nanofluids with a magnetic effect. The behaviour of micro-convective nanofluid flow is gaining interest nowadays. Taxis and kinetics are two kinds of processes by which micro-convection can happene. They respond to stimuli from many sources, like light, chemical reactions, and temperature. Houseflies are attracted by the light while mosquitoes move away from light, this behaviour of insects is known as positive and negative taxis, respectively.

Our aim through this research is to examine the mathematical model containing two-dimensional MHD steady stream, mass, and heat transport incompressible micro-convective nanofluids with Arrhenius activation energy. A suitable similarity transformation has been used to non–dimensionalize the system of non-dimensional ordinary through adequate transformations. The converted nonlinear coupled ODEs have been solved by operating the Spectral Quasi-linearization technique numerically. The impacts of key parameters have been investigated diagrammatically.

MATHEMATICAL MODEL

Here a two-dimensional magnetohydrodynamics thermo-convection nanofluid flow with Arrhenius activation energy has been taken into consideration. The mathematical form is $u_w(x) = cx$, where is $c > 0$ (Fig. **1**).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_f(1+N^2)}u + (1 - C_\infty)\rho_f g\beta_T(T - T_\infty) - \frac{(\rho_p - \rho_f)}{\rho_f}g\beta_C(C - C_\infty) \tag{2}$$

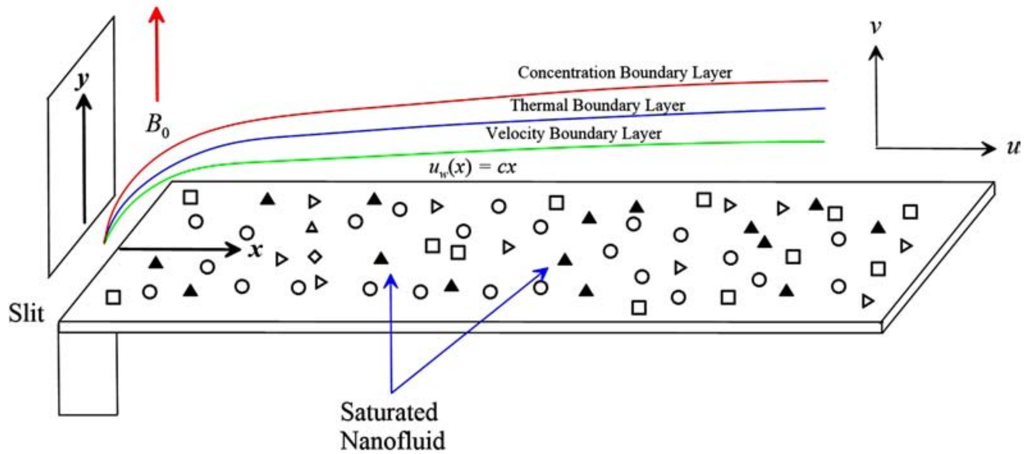

Fig. (1). Physical model.

The governing equations of fluid flow for this present model are defined as follows:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho_f C_p}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\left(\rho C_p\right)_f}\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}$$
$$+Q(T - T_\infty). \quad \textbf{(3)}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} - k_r^2\left(\frac{T}{T_\infty}\right)^n \exp\left(\frac{-E_a}{kT}\right)(C - C_\infty) + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \quad \textbf{(4)}$$

The suitable boundary conditions are considered from Eqs. (1)-(4) as follows:

$$u = cx\,, \quad v = 0, \quad -k_f\frac{\partial T}{\partial y} = h_f(T - T_\infty), \; D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y},\!=\!0, \quad \text{at } y = 0\,.$$

$$u = 0\,, \quad T = T_\infty\,, C = C_\infty \text{ at } y = \infty\,. \quad \textbf{(5)}$$

Eqs. (1) and (2) represent the momentum along $x - axis$ and $y - axis$, respectively. σ and ρ_f , ρ_p , are the electrical conductivity, and density of fluid and nanoparticles, respectively. γ , and σ^* are the Stephan-Boltzman constant and microorganism expansion coefficient, respectively.

CHAPTER 4

Study of Oblique Nanofluid Flow Past a Stretching Sheet

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Abstract: The analysis of impinging oblique nano-fluid past a stretching sheet is done in the current communication. The nanoparticles' behaviour in fluids for convective heat transfer has been extensively studied. We assume that the fluid and the stretching velocity change linearly. In this paper, we used a cutting-edge Genetic Algorithm technique to better understand how a nanofluid's thermal characteristics and heat transport vary over time. Additionally, the impact of a change in the fluid's impinging angle is investigated. The temperature and concentration curves, skin friction coefficient, and Nusselt number have all been observed and graphically displayed.

Keywords: Genetic algorithm, Heat transfer, Nanofluids.

INTRODUCTION

The traditional transport of heat in nanofluids has recently gained attention as a coeval topic. Choi was the first to use the term "nanofluid" [1]. A base fluid and nanoparticles are combined to form a fluid called a nanofluid. With their underlying fluids, they have extremely high thermal conductivity. Nanoparticles have a diameter that falls between 1 and 100 nm.

Typically, the metals (Al, Cu), oxides, carbides, nitrites, and carbon nanotubes make up the nanoparticles employed in nanofluids (allotrope of carbon nonmetal). When combined with other substrates, nanoparticles have the inherent ability to improve thermal conductivity. Base fluids include conductive liquids like water, oil, ethylene glycol, *etc*.

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The conventional heat transfer fluids' thermal conductivity is insufficient to meet modern industries' demands for cooling rates. Considered as cutting-edge technology to ensure nuclear safety is a nanofluid coolant with improved thermal performance. In order to guarantee a functional improvement in heat transmission, nanoparticles commonly possess up to a modest 5% volume fraction of nanoparticles. An experimental investigation disclosed that a slight volumetric percentage of nanoparticles significantly increased the thermal conductivity by 10–50% with an impressive mastery of the fluid.

Buongiorno [2] provides a thorough overview of convective transference in nanofluids. He discovered that the slip velocity can be expected to add up to the nanoparticle's velocity (relative velocity). Following his reasoning, he concluded that, only Brownian motion and thermophoresis are momenta. Based on the mechanics of the relative velocities of the nanoparticle and base fluid, Buongiorno suggested a new model.

Various processes involve pushing when an incompressible viscous fluid flows across a stretched surface. Modern academics have been drawn to the current topic because of its amazing applications. Crane [3] investigated the effects of applying uniform stress to a 2-D continuous flow and concluded that the stretching of an elastic sheet changes linearly from a constant point.

The stagnation point flow approaching the extending surface was examined by Gupta and Mahapatra [4]. Singh *et al*. [5-8] investigated how magnetic parameters and radiations affected several aspects of the stretched sheet. Recently, the study of magnetohydrodynamic flow is of utmost importance. The deformation of a fluid-containing vessel's wall causes magnetohydrodynamic flow to occur. A unique substance with both liquid and magnetic qualities is known as magnetic nanofluid. By changing the magnetic field, it is possible to regulate several physical characteristics. Hamad [9] investigated the convective flow of nanofluid *via* a semi-infinite vertical sheet. In a nanofluid, Bachok *et al*. [10, 11] investigated the boundary layer flow toward a stretching sheet.

Genetic algorithm, the main paradigm of evolutionary computation, is an optimization technique. It relies on Darwin’s theory of the “survival of the fittest” that drives biological evolution. John Holland [12] was the inventor of the genetic algorithm who described the technique in his book “Adaptation in Natural and Artificial Systems”. Lots of researchers and scientists since then have contributed to this area with varying degrees of success. McCall [13] has explained that the genetic algorithm is an active and flourishing area that has the potential to reach the zenith. It can explain even the complex phenomenon that was out of reach for the earlier theories. Chaiyartna and Zalzala [14] have reviewed the developments in this area. A technique for resolving optimization issues based on the idea of natural selection is the genetic algorithm. The genetic algorithm employs three important processes, first, the selection where the individuals are randomly selected from the available parents, second the crossover where the selected parents combine to form the next generation and the third mutation in which random changes occur at the parent level to form children. The population formed after successive iterations evolves towards an optimized solution. It has an edge over the standard optimization algorithms in various linear and non-linear physical problems, such as heat and fluid flow because of its highly modular. In recent days, these flows are either studied by finite difference method with shooting technique or other optimization techniques. These obsolete methods falter at the front that they require an initial guess for the function to be optimized. Soft computing algorithms are the best and most reliable alternative for getting the optimum design in fluid dynamics.

FORMULATION OF THE PROBLEM

A stretching sheet having invariable temperature T_w is the physical issue being considered here on which the viscous nanofluid is incident obliquely means at a certain angle of incidence (Fig. **1**). The nanofluid has characteristics of incompressibility, steady flow in two dimensions in the upper half plane and good electrical conductivity. The stretching surface is positioned along the x-axis and in the plane y = 0. A magnetic field of transverse nature is applied on this nanofluid which possesses the non-uniform velocity of the form of $u_w = Ax$ where A is a positive constant having dimension (time) −1.

The Joule heating effect, the magnetic field that can be created in the due course of flow, and the dissipation caused by the fluid's viscous nature were some of the aspects that we overlooked while examining this physical issue. The various conservation equations strictly adhering to above stated assumptions may be written as under-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \textbf{(1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_b}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma\frac{u}{\rho}B^2 \quad \textbf{(2)}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_b}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \textbf{(3)}$$

CHAPTER 5

Effect of Casson Nanofluid's Outer Velocity with Melting Heat Transmission beyond a Stretching Sheet

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Abstract: An analysis was conducted to observe the collective consequence of the melting procedure across a stretching sheet in a Casson nanofluid with outer velocity. The outcomes of melting heat parameter (M) and Casson parameter (β) are shown in this analysis. The leading PDEs are moulded in ODE with similarity variables. The numerical results of moulded ODE are given by the RKF scheme with shooting procedures. Important parameters of the nanofluid are examined and graphically displayed. Tables are used to display the Sherwood number, Nusselt number, and skin friction coefficient based on different nanofluid characteristics. The development of cooling systems that raise the heat transmission belongings of nanofluids and increase engineering potential will be greatly aided by this research.

Keywords: Casson nanofluid, Melting heat transmission, Outer velocity.

INTRODUCTION

The classic boundary layer issues are expressed as PDEs. It is commonly recognized that PDEs are used in science and technology fields. The studies [1, 2] broadly explain their foremost belongings on real-world challenges.

Based on the investigational data, nanofluids are more thermally conductive than ordinary fluids. Eastman and Choi developed nanofluids back in 1995 who argued that the existence of scattered nanoparticles increased the pure fluid's true thermal conductivity [3]. In nanofluids, the depth coverage of convective transference was provided by Buongiorno [4].

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It is common knowledge that physical transformation takes on melting by heating which results in the phase transition of materials from solid to liquid. The melting process has served as the subject of incredibly challenging theoretical and practical research for many years. Real-world applications that include welding and casting in manufacturing, defrosting frozen ground, conserving heat energy, heating, and cooling.

Consequently, melting has been used in a few experiments. The melting of ice in an airstream that is heated was studied by Roberts [5]. The relationship between melting and diffusion transmission of heat was primarily studied [6]. In laminar flow, the transmission of melting heat was studied by Epstein and Cho [7]. The use of natural convection for melting was studied by Sparrow *et al*. [8]. Only a few researchers [9-13], have recently researched melting heat transmission in boundary layer problems with various angles.

Therefore, heat transmission *via* melting in a Casson nanofluid flow past a stretched sheet with an outside flow was investigated in this work. The method of Newton-Fehlberg iteration is used to crack the transformed nondimensional ODE. Detailed descriptions of the pertinent flow factors are provided.

MATHEMATICAL FORMULATION

Consider a stretched sheet that is implanted through outside velocity and melts while being continuously traversed through a 2-D incompressible Casson nanofluid, as depicted in Fig. (**1**).

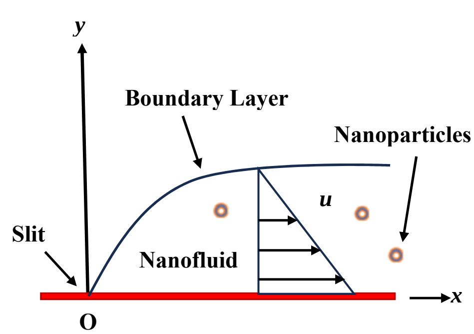

Fig. (1). Schematic diagram.

The governing equations are [14]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U(x)\frac{\partial U(x)}{\partial x}+v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \quad \textbf{(2)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \quad \textbf{(3)}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \quad \textbf{(4)}$$

Restricted surroundings designed for the flow are:

$$v=0,\quad u=u_w,\ k\frac{\partial T}{\partial y}=\rho[LH+C_s(T_w-T_0)]v(x,0),\ T=T_w,\ C=C_w \quad at\ y=0 \quad \textbf{(5)}$$

$$u\to U(x),\qquad v=0,\qquad T\to T_\infty\qquad C\to C_\infty\qquad as\ y\to\infty \quad \textbf{(6)}$$

In this case, thermal conductivity, solid surface temperature, latent heat, heat capacity and density are represented by variables K, T_0, LH, C_s and ρ

The dimensionless and similarity quantities are:

$$\psi=\sqrt{xvU(x)}\,f(n),\quad \eta=\sqrt{\frac{U(x)}{xv}}y,\quad \theta(\eta)=\frac{T-T_w}{T_\infty-T_w},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-T_\infty}, \quad \textbf{(7)}$$

Stream function $\psi(x,y)$ is given as:

$$v=-\frac{\partial\psi}{\partial x},\ u=-\frac{\partial\psi}{\partial x} \quad \textbf{(8)}$$

Using Eqs. (7) and (8), Eqs. (2-4) become:

$$\left(1+\frac{1}{\beta}\right)f'''+(\lambda^2-f'^2)+ff''=0 \quad \textbf{(9)}$$

$$\theta''+P_r\{N_b\phi'\theta'+f\theta'+N_t\theta'^2\}=0 \quad \textbf{(10)}$$

$$\phi''+\frac{N_t}{N_b}\theta''+Le\phi' f=0 \quad \textbf{(11)}$$

The restricted surroundings are as follows:

$$B\theta'(\eta)+P_r f(\eta)=0,\ f'(\eta)=0\ \ \theta(\eta)=0,\quad \phi(\eta)=0 \text{ at } \eta=0 \quad \textbf{(12)}$$

CHAPTER 6

Dual Solutions of Magneto-Radiative Ag-Water Nanofluid Slip Flow Across the Porous Medium Due to a Permeable Contracting Surface with Heat Generation: Stability Analysis

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Abstract: The current study looks at the heat transfer of a magneto-radiative nanofluid over an exponentially contracting permeable sheet in the existence of heat generation with numerous slip boundary constraints. It also looks at the stability and duality of solutions. Here, a water-based fluid with silver (Ag) nanoparticles is employed. Before being computed by bvp4c in the Matlab program, the governing nonlinear partial differential equations are converted into dimensionless nonlinear ODEs *via* a similarity transformation. Due to the contracting surface scenario, a dual-nature solution can only be found if a sufficient suction value is used. We may infer from stability investigation that the first one is stable while the second one is unstable. A stable solution makes sense with the least positive eigenvalues, but a lower unstable solution indicates negative eigenvalues. The Nusselt number and the skin-friction factor may be improved by increases in the silver nanoparticle's solid volume percentage. The least eigenvalue converges to zero as the suction and contracting surface parameters reach their critical values. Nanofluids that are magnetic and radiative can be used to create beautiful and effective electromagnetic devices, including in the areas of clothing, paper, plastics, food colorants, cars, cancer therapy, medicines, ceramics, soaps, and paints.

Keywords: Dual solutions, Exponentially contracting surface, Heat generation, Magnetic field, Nanofluid, Porous medium, Stability analysis, Thermal emission.

INTRODUCTION

Energy conservation in heat transmit systems is a critical issue in a variety of cutting-edge industries and mechanical applications. Conservative fluids such as water, polymeric solutions, biofluids, glycols, and lubricants, have been employed as heat transmit fluids for many years. Due to their poor thermal conductivity, they have a constrained ability to transport heat. The conductivity of nanofluids, which

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is a combination of nano-sized particles-which may be metal oxides, metals, polymers, or other materials of size 1-100 nm-and the host fluid is greater, increasing the rate of heat transmission. Choi and Eastman [1] first conducted several studies to validate their novel, ground-breaking concept. Applying metal flakes with high thermal conductivities requires only a small amount of the particles. Studies on heat transmission in nanofluid movement over contracting surfaces have gained popularity due to their practical applications in a variety of technical and industrial disciplines, including the manufacture of paper, artificial fiber and polymer extraction, extrusion, and glass blowing. Researchers have also begun to apply stability analyses to issues with dual contracting-surface solutions, with one being steady and the other unstable. The first stability study to verify the stable solution was performed by Merkin [2]. When calculating eigenvalues, he discovered that nonnegative eigenvalues had stable while the negative eigenvalues had an inconsistent solution. Later, a large number of scholars performed stability analyses for dual solutions in mixed convection movements or extending or contracting surfaces. Weidman *et al*. [3] investigated the impacts of transpiration on movement on moving sheets. They discovered multiple findings and stated that, when blowing and suction effects are taken into account, the spectrum of dual solutions either diminishes or grows. Dero *et al*. [4] investigated slip impacts on the unsteady transport of nanofluid across a contracting surface while using several findings. Ghosh and Mukhopadhyay [5] experimented with nanofluid movement through an exponentially penetrable shrinking sheet in the existence of slip, with a dual nature of solution and stability analysis. The creation of entropy by nonlinear heat emission on a non-Newtonian nanofluid in magnetohydrodynamics was studied numerically by Bhatti *et al*. [6] using a permeable contracting sheet. Tiwari *et al*. [7] studied MHD flow across a non-linearly extending or contracting sheet using water-based nanofluids made of silver (Ag) and titanium (TiO_2). Triple solutions were discovered by Lanjwani *et al*. [8], who looked into the heat transmission properties of mixed-convective Ag-H_2O nanofluid flow across a permeable contracting and extending sheet. Fe_2O_3 and Fe-water base nanofluid boundary layer movement and heat transmission across an extending or contracting sheet with a radiation impact were studied by Lanjwani *et al*. [9] in terms of stability. The stability study for a hybrid nanofluid movement for the effects of Joule heating and viscosity across a contracting plate was conducted by Rasool *et al*. [10]. The stability study of radiative magnetic hybrid nanofluid slip movement caused by an exponentially extending or contracting penetrable sheet with heat production was recently the topic of research by Mandal and Pal [11].

The mixing of electromagnetism with fluid mechanics, or the effects of magnetic fields on electrically conducting fluids that can regulate the rate of heat transmission and flow in a system, is studied as magnetohydrodynamics. A magnetic field causes a fluid to travel across it, creating an electric current. In a variety of engineering and industrial applications, such as in nuclear reactors, magnetic mixers, MHD power generators, chemical reactions, metal casting, petroleum industries, and metallurgical processes, this electric current bears the potential to significantly affect both the flow and temperature behavior of the fluid. Using suction and injection, heat generation and absorption, and a heat-permeable extending and contracting surface, Chaudhary and Kanika [12] studied the influence of hydromagnetic forces on the movement of radiative nanofluids having stagnation points. The unsteady MHD stagnation point movement for nanofluid *via* the penetrable contracting surface has been examined by Roy and Pop [13]. Casson nanofluid across a vertical exponentially diminishing sheet having an angled magnetic field was examined by Ishtiaq *et al*. [14]. A penetrable stretched sheet with heat emission and viscous dissipation was the object of investigation in magnetohydrodynamic nanofluid movement by Muntazir *et al*. [15]. Using slip boundary constraints and entropy generation, Mandal and Pal [16] studied the radiative magneto single and multi-walled CNT/H_2O hybrid nanofluid flow.

Many recent studies have been published and are fully available on the subject of heat emission over the motion of the boundary layer. Convective flows in nanofluids are also extensively used in engineering, such as in geothermal systems, groundwater pollution, nuclear waste storage, crude oil extraction, thermal insulation, agriculture, heat exchangers, refrigerators, hybrid engines, microelectronics, cancer therapy, vehicle air conditioning systems in transportation, *etc*. The flowing movement of micropolar nanofluid *via* the stretched surface having a heat sink or source (non-uniform) was shown by Pal and Mandal [17] using thermal emission and MHD processes. The effects of heat emission on nanofluid movement bound by partial slip constraints were investigated by Rusdi *et al*. [18]. Azam [19] examined the importance of heat flow and non-linear thermal emission over an Arrhenius-activated magnetohydrodynamic Maxwell nanofluid. An issue with the calculation of thermal emission in nonlinear mode over magneto nanofluid movement by entropy production was shown by Farooq *et al*. in their study [20]. Alzahrani [21] investigated the impact of thermal emission on heat transmission in a Casson nano liquid containing suction flowing in a plane wall jet under slip boundary constraints.

CHAPTER 7

Spectral Quasi-Linearization Method for Thixotropic Nanofluid Passing through a Stretching Surface with Activation Energy

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Abstract: The primary goal of the current investigation is to understand the thixotropic nanofluid's magnetohydrodynamic (MHD) nonlinear convective flow. The implementation of appropriate transformations allows for the transition of sets of ordinary differential equations from partial differential systems. Spectral Quasi-Linearization Method (SQLM) is applied to solve these ODEs. Further study has been performed in order to establish a model for nanomaterials containing thermophoresis phenomena and Brownian motion. Graphs are used to depict and discuss the effects caused due to numerous quantities of fluid flow, temperature, environment, and nanoparticle concentration. In addition to this, the local Nusselt number, the skin friction coefficient, as well as the Sherwood number's numerical values, are determined and examined. Some interesting observations on velocity, heat, and concentration have been observed due to the variation made in thixotropic parameters and other allied parameters such as thermophoresis and Brownian motion.

Keywords: Newtonian heat and mass conditions, Magnetohydrodynamics (MHD), Spectral quasilinearization method (SQLM), Thixotropic nanofluid.

INTRODUCTION

There are currently a large number of applications for non-Newtonian materials in technology and business. Due to numerous applications in physiology, medicines, oil reservoirs, coating of wires, grease, fibre technology, physiology, food products (such as crystal formation, milk, and apple sauce), *etc.*, researchers and scientists have been motivated to consider this. Fetecau *et al*. [1] examined the energy balance for an Oldroyd-B fluid flowing through a straight plate under a time-varying shear stress. Wang and Tan WC [2] analysed the stability of a convective double-diffusive Maxwell fluid in a porous medium. Despite these challenges, most

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researchers are always attempting to analyze the behaviour of different non-Newtonian fluids from numerous perspectives. Shear thinning and thixotropic fluid vary in such a way that the latter demonstrates viscosity reduction in a liquid which is shear thinning in nature with an increase in shear rate and the former indicates viscosity degradation over time. One can examine a few studies on thixotropic substances. Qayyum and Hayat [3] examined the MHD nonlinear thixotropic nanofluid flow under Newtonian heat, mass conditions, and chemical reactions. Olubode and Nehad [4] explored the examination of MHD thixotropic nanofluid bioconvection flow past a vertical surface in the presence of gyrotactic bacteria and nanoparticles. Mehta and Chouhan [5] examined the MHD flow of nanofluids through a vertical channel while being exposed to radiation, heat, and porous media. Nanofluid is a word used to describe the diluted suspension of fibres and nanometer-sized particles in a fluid. In comparison to typical fluids, the mixing of nanoparticles altered the physical characteristics of the liquid, such as density, thermal conductivity, viscosity, *etc.* Dharmaiah, and Dinarv [6] analysed the Buongiorno non-homogeneous two-component model in the presence of activation energy for Howarth's wavy cylinder. Shafiq *et al*. [7] used a machine learning algorithm to establish the implication of bioconvective flow of MHD thixotropic nanofluid traversing a vertical surface. Waqas *et al*. [8] analysed the joule heating impacts and nonlinear convection in the flow of magneto-thixotropic nanofluid under convection by a heated variable over a thicked surface. The use of nanoparticles is widespread in a variety of fields, including smart computers, renewable energy, solar cells, catalysis, materials, electronics, optics, and many more. Currently, there are two categories in which the idea of energy transmission in nanofluids might be placed. One category of nanofluids was assumed to be composite, composed of nanoparticles, and shielded by nanolayers. Anantha [9] analysed MHD nonlinear radiative slip motion across a stretching sheet of a non-Newtonian fluid numerically examined in a porous medium. Madiha, *et al*. [10] numerically analysed the unsteadiness of the three-dimensional Williamson fluid-particle flow when suspended under MHD thermal radiations. Kataria *et al*. [11] established the impact of nonlinear radiation on fluid flow for MHD with optimised entropy. The studies contain some inquiries into this idea. According to the alternative idea, micro-mixing results from a combination of particle convention and Brownian motion, which determines how well a nanofluid conducts heat. In these models, particle dynamics is taken into account. Through the use of this theory, the particle interaction in nanofluids can be seen. In general, four categories of heating mechanisms define the boundary temperature fields. These categories include (i) arbitrary or constant heat flux on the surface; (ii) conjugate conditions; and (iii) arbitrary or constant wall temperature, in which temperature has been moved to a convective fluid passing through a boundary surface along with a limited specific heat capacity and thickness. The merging temperature is determined by the underlying assumptions of the system, specifically the thermal conductivity of solid and liquid, which is unknown beforehand. Conjugate convective flow is the primary characteristic of Newtonian heating and the rate of heat transfer from the boundary surface, which has a restricted specific heat, is related to the local surface temperature. Heat exchangers, solar radiation, storage of thermal energy, the petroleum sector, and other fields are among Newtonian heating's applications. Das [12] established in a rotating reference frame, that there is an impact of thermal radiation and chemical reaction over the mass and heat transfer of MHD micropolar fluid flow. Jamil [13] established fractional derivatives with heat radiation and chemical reactions which are used to represent MHD Maxwell flow.

In this article, the convective flow of a non-linear thixotropic nanofluid is examined in the presence of a magnetic field, Newtonian heat, and mass conditions. This paper is extended in which the flow of thixotropic nanofluid passing through a stretching sheet is examined under activation energy and thermal radiation. Here spectral quasi-linearization method is used. This Brownian motion, heat absorption/generation, and thermophoresis are studied. Quasilinearisation is used to linearize nonlinear ordinary differential equations. The use of a spectral method in order to solve the linearised equations is called the spectral quasi-linearisation method (SQLM). The SQLM assumes that there is little variation in the solution's approximation at the current iteration level and the one from the prior iteration. Also, the derivatives at the next iteration levels are considered to differ just slightly.

MATHEMATICAL FORMULATIONS

We consider a stretching sheet as the path of a continuous non-linear, two-dimensional, and convective flow of thixotropic nanofluid. The stretched surface's flow is contained inside the domain $y \geq 0$. The stretching velocity of the sheet is $u_w(x) = cx$ (c being a non-negative constant). The amplitude of a constant magnetic field B_0 is parallel to the $y -$axis (perpendicular to the stretched surface) (shown in Fig. **1**).

The induced magnetic field has been disregarded since the Reynolds number is thought to be negligible. When thixotropic fluid flow equations are subjected to standard boundary layer approximations, they provide:

CHAPTER 8

Effect of MHD Viscous Nanofluid Flow in the Presence of Internal Heat Generation

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Abstract: In this current study, we investigated the boundary layer steady nanofluid flow which is viscous. Utilizing the spectral quasi-linearization method (SQLM), the mathematical model has been solved. To analyze the convergence of the numerical method, we calculated the residual errors that are approximately less by [10^{-8}]. The local Sherwood number, the coefficient of the rate of heat transfer, and the local drag force coefficient along with various flow parameters are analyzed numerically and graphically. The influence of thermophoresis and Brownian motion has also been discussed briefly.

Keywords: Brownian motion, Heat generation, Nanofluids, Thermophoresis.

INTRODUCTION

Nanofluids are such a fluid in which nano-sized particles are suspended in a base fluid to such an extent that no sedimentation can take place. Nanofluid is an adhesive solution of nano-molecule in the base fluid. All kinds of metal, carbon nanotubes, carbides, oxides, *etc.* are used as nanoparticles, and water, oil, ethylene glycol, and others are used as the base fluid. Nanofluids are known for their unprecedented prominences which are inequivalent from the base fluid and involve thermophysical characteristics like density, specific heat, thermal conductivity, viscosity, *etc.* The thermophysical features of fluid are changed after adding nanoparticles to it. Thence nanofluids are used for their improved thermal features as cooling in heat conduct tackle like heat exchangers, electronic coolant systems, and convectors. A magnetohydrodynamic field produces currents in a moving conducting fluid flow that produces energy and generates a magnetic field effect. Most industrial procedure entangles the stream of reactive liquids. Considering the

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nanofluids boundary layer stream which is incompressible and viscous in nature, we have investigated heat and mass transfer throughout the stretching lamina. Such streams are confronted in industrial procedures like polymer extrusion, plastic, metal ejection, and glass blowing studied by Makinde and Sibanda [1], Hayat *et al.* [2], and Sohut *et al.* [3].

The rate of cooling and the chemical reactions in manufacturing industries measure the grade of products studied by Yongjae [4] and Boekel [5]. In a chemical reaction in the food product industry in a significant area, heat can be produced/absorbed by a chemical transformation examined by Olanrewaju *et al.* [6], so controlling heat transportation in a system is very important.

Bhattacharyya [7] applied an effective magnetic field to control the heat transportation rate. Nanofluid is considerably efficient in transporting heat and mass through the fluid flow. It is very useful in the industrial area and medical sector to apply Nanofluid instead of ordinary fluid explained by Abu-Nada [8] and Mohammed *et al.* [9]. Nanofluids are the compositions of nano-sized particles and base fluids, typically metal or metal-oxide, carbides, SWCNTs, or single-wall carbon nanotubes, and MWCNTs or multi-wall carbon nanotubes are mixed in base fluids like water, oil, ethylene glycol, *etc.* Application of nanofluids occupies an extensive area in solar water-heating, cooling techniques, electronics equipment, and the enhancement of the efficiency of diesel-based generators examined by Hussein and Hussain [10], Hayat *et al.* [11] and Mahanthesh *et al.* [12].

The issue of the production of heat in the system has been exhibited to be very concerning, so giving attention to these physical phenomena of heat production is considerable. The fluid velocity attains the extreme value in case of no thermal radiation. Palani *et al.* [13] scrutinized the same type of investigation excluding energy transportation, we detect that a simultaneous study of velocity profile along with energy and mass transportation may be much more effective as they are correlated to each other.

Mlamuli *et al.* [14] found that the sequel of the Brownian motion parameter and the thermophoresis parameter have a revoking reaction on the velocity profile and concentration profile, whereas both raise the fluid temperature. Sarkar *et al.* [15] examined that the momentum boundary layer reduces as the magnetic parameter increases.

Mondal *et al.* [16] investigated the power-law fluid and solved the numerical model by using the spectral quasi-linearisation method. The impact of thermal radiation on nano-fluid flow with higher-order chemical reactions was studied by Sarkar *et al.* [17].

The current study aims to investigate the momentum, energy, and mass transportation of a nanofluid. Important factors like internal heat production, the effect of the magnetic field, dissipation, buoyancy force, and thermophoretic effect have been taken into account to identify their impact on the fluid velocity profile, thermal energy, and mass transportation characteristics. Convective boundary conditions are considered in this present study. Here spectral quasi-linearization method is used to solve the model equations.

FORMULATION OF THE PROBLEM

Along the way of the mainstream of the lamina, $x - axis$ has been considered. The direction of $y - axis$ is perpendicular to the lamina with velocity components u, v along $x - axis\ and\ y - axis$, respectively see (Fig. **1**).

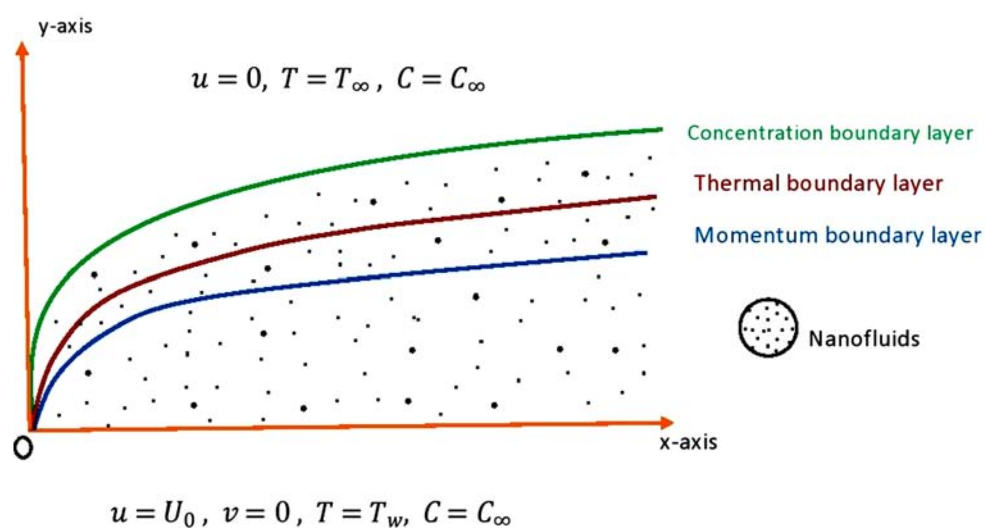

Fig. (1). Schematic diagram.

The ruling equations of a two-dimensional stream considering the boundary layer approximation are presented in this fashion:

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Sabyasachi Mondal

Dr. Sabyasachi Mondal is an Associate Professor in the Department of Mathematics at North-Eastern Hill University, Shillong, Meghalaya, India. He has obtained his B.Sc. (H) in Mathematics, M.Sc. in Applied Mathematics, and Ph.D. in Computational Fluid Dynamics from Visva-Bharati Santiniketan, West Bengal, India. He worked as a 'Postdoctoral Research Fellow' at the University of KwaZulu-Natal, South Africa.

His areas of research interest include Computational Fluid Dynamics, Heat and Mass Transport, Nanofluid flow, etc. Along with his 12 years of teaching experience, he has more than 90 international publications in different reputed journals, books, and conferences in his credit. Also, Dr. Mondal published one Indian patent. Dr. Mondal delivered invite talk in different Universities and colleges. He is an editorial board member in different international journals and a reviewer of various reputed journals in Elsevier, Springer, Wiley, Taylor & Francis, Bentham Science etc.

Dr. Mondal has received various prestigious funds and fellowships like UKZN PDF fellowship, Claude Leone Postdoctoral fellowship, NRF travel grants, etc. Dr. Mondal has research collaborations with various eminent scientists in India and abroad, like South Africa, Botswana, Malaysia, Iran, etc. His research has led to him being selected on the 'Top 2% Most Influential Scientists List' published by 'Standford University for the year 2022-2023.