

Numerical Methods and Implementation in Geotechnical Engineering — Part 1



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Numerical Methods and Implementation in Geotechnical Engineering – Part 1

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PREFACE

For most of the geotechnical problems, particularly those related to real life problems, analytical solutions are usually not available. For both research and practical applications, numerical methods and computer programs are required for many cases. In the recent forty years, many numerical methods have evolved for various kinds of engineering problems. Engineers are now well adapted to the uses of different computer programs for the solution of engineering problems. There is however a major drawback in the current engineering practice in that most of the engineers are not familiar with the basics of the numerical methods, the methods of implementations and the limitations of the numerical methods/programs. In fact, to a certain extent, the methods of implementations and the limitations of the numerical methods are related. In many internal studies using different commercial numerical programs, the authors sometimes found noticeable or even completely different results with different programs or the same program with different default setting for a given problem, and this situation is not uncommon. For a problem with unknown solution, how an engineer assess the acceptability of the computer results is a difficult issue that needs serious attention. In several technical meetings in the Hong King Institution of Engineers, the authors have discussed with some engineers about the appreciation of the limitations of the daily-used engineering programs. If two computer programs can produce significantly different results, how an engineer determine the acceptability of the results actually require deeper knowledge about the basics of the numerical methods and implementations. Interestingly, the authors like to ask the students a question “Different answers can be obtained from different commercial programs. Which results should be accepted, and why should those results be accepted?”. In general, the authors challenge the students (undergraduate and graduate students) every year for this question, and virtually this question is never answered properly. The problems in the assessment of the numerical results will also be discussed in this book, which is seldom addressed in other books or research papers.

The authors have participated in different types of geotechnical research and consultancy works in different countries, and has written a book *Frontier in Civil Engineering, Vol. I, Stability Analysis of Geotechnical Structures*, which is well-favored by many students, engineers and researchers. Most of the books on numerical methods seldom address the actual procedures in numerical implementations, but many postgraduates actually need to develop computer programs to consider special constitutive models, loadings, numerical methods, boundary conditions and other effects. In view of the limitations of most of the books at present, the authors would like to write a new book on numerical methods and the implementations based on their previous works, and this new book should be useful for senior undergraduates, postgraduates, engineers as well as researchers.

In this book, finite element method, optimization method, plasticity based slip line method, limit analysis method, distinct element method, Smoothed-Particle Hydrodynamics Method, Spectral Element Method and Material Point Method will be introduced. The present book will not cover dynamic problems which is a big topic, and hopefully this will be covered later by the authors in another book. The authors will also try to explain the methods of implementation for some of these methods through sample computer programs. Sample programs are given and discussed to assist students in developing programs for their own uses. These programs are not meant to be efficient or up-to-date, but will help the students in learning about the implementation of some numerical methods. This book should not be taken as a classical textbook, as the authors do not intend it to be. There are many new contributions to numerical methods in geotechnical engineering over the last 30 years, and many topics can be covered by individual books for detailed discussion. There is also no way for the authors to

cover all numerical methods in details in this book. This book is a basic introduction to some more commonly used numerical methods in geotechnical engineering which have been used by the authors for teaching and research, with the discussion of some common commercial program problems, programming techniques and applications.

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CONSENT FOR PUBLICATION

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CONFLICT OF INTEREST

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Introduction

Abstract: This chapter is an introduction to the requirement and the various problems that will be encountered during numerical modelling in geotechnical engineering. A large scale tunneling work in Australia will be used to illustrate the necessity to use numerical methods in some real life engineering problems. After that, the authors will introduce a series of numerical problems that may be encountered during the use of commercial engineering programs. Such problem cases may arise from various sources, and engineers are strongly advised to understand the basic principle of each commercial program and to assess the program output with care before accepting the results of analysis. Finally, some of the more important governing differential equations for geotechnical problems are discussed.

Keywords: Errors, Finite element, Governing differential equations, Modelling, Numerical methods, Slope stability, Tunneling.

1.1. INTRODUCTION

For most of the geotechnical problems, particularly those related to real life problems, analytical solutions are usually not available. The authors have carried out many research works and large scale practical projects, and in general, most of the works are complicated in both geometry, applications of loadings, construction sequences, material behavior, ground water conditions as well as other factors. As a good illustration, the construction of the Airport Link project in Brisbane at Australia is a good example (Cheng *et al.* 2019). The project is located beneath the railway embankment of the North Coast Railway line adjacent to Kalinga Park, and the site comprises a thick layer of soft clay. The Airport Link, which is one of the most complex roads and tunnel engineering feats in Queensland's history, will be the first major motorway linking Brisbane city to the northern suburbs and airport precinct. The Link is a 6.7km toll road, mainly underground, connecting the Clem 7 Tunnel, Inner City Bypass and local road network at Bowen Hills, to the northern arterials of Gympie Road and Stafford Road at Kedron, Sandgate Road and the East West Arterial leading to the airport. At one of the project sites, the tunnel section under the QR railway embankment at Toombul is constructed by box jacking technique. The significant size of the

launch box requires 85,000m³ of spoil to be excavated under the railway embankment. Headwalls, canopy tubes and sidewall nails are constructed to retain the railway embankment for the excavation of the jacking shafts. The challenging ground conditions and requirements for the present project require the combinations of innovative ground support, construction methods and detailed and realistic analysis for the proper execution of the works. In this project, the site is mostly composed of soft clays which are susceptible to ground settlement problem during construction, and a typical section is shown in Fig. (1.1). The SPT value for the soft clay is less than ten, whereas the CPT friction ratio for soft clay ranges between 2% and 4% with a mean pore pressure of approximately 0.12 MPa (see also Table 1.1). The SPT value for the firm clay is approximately 20, whereas the friction ratio for firm clay ranges between 4-8% with a mean pore pressure of approximately 0.38 MPa. The railway has to be maintained in operation during the whole construction to ensure the transportation, and the settlement of the soft clay must be maintained at a low level with minimal disturbance to the railway track. This is technically a very difficult problem, and the original construction proposal is to inject large amount of grout into the ground to stabilize it prior to excavation. However, the cost of the original scheme is extremely high so that a more economical alternative is considered. Ground improvement works underneath the QR railway embankment are hence required for the stability consideration during box jacking stages. A trapezoidal jet grout block constructed immediately behind the headwall is used as a gravity type retaining wall to reduce the earth pressures on the piled headwall. A smaller jet grout block is provided at the north west of the final jacked box location and is used as an anchorage to the northern sidewall nails. A low strength grout wall is installed west of the railway to provide a water cut-off for the TBM launch box. The grout wall is also used in the jacking scheme design to provide adequate anchorage to the geonails at the receiving pit side, eliminating an approximate 10m length of nail with significant time and cost savings. The resulting 'nail anchored' western grout wall can then be used to maintain slope stability, enabling initial excavations in the cut and cover receiving pit to commence early.

Table 1.1. Average properties of ground soil (Young's modulus determined from dilatometer, vane shear and CPT tests).

Soil	Undrained Shear Strength. (kPa)	Young's Modulus. (MPa)	Water Content. (%)	Plasticity Index
Soft clay	20	6	57	25
Firm clay	37	20	46	45

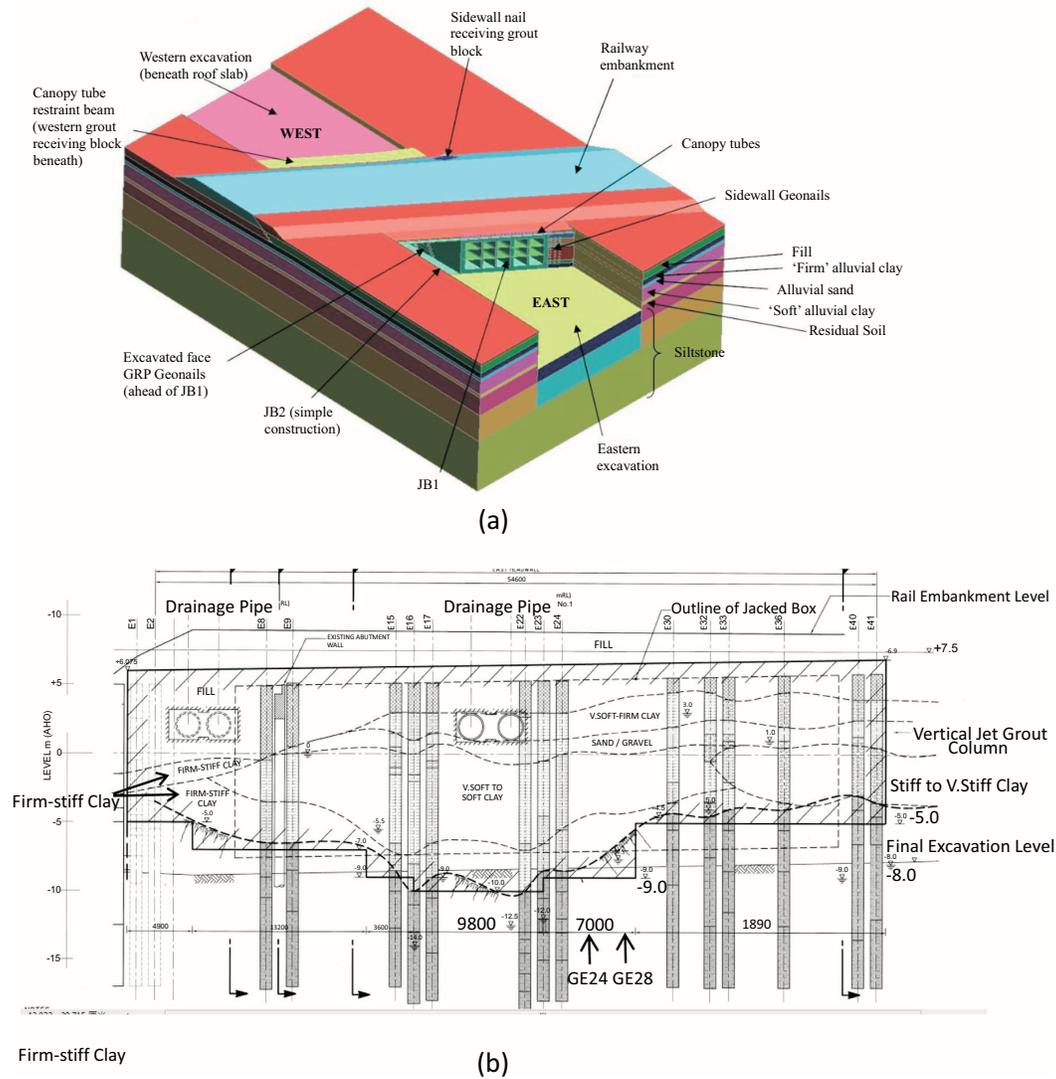


Fig. (1.1). (a) Geological condition for the tunnel project in soft clay, (b) a typical section of the tunnel work.

In order to optimize the ground improvement design, a combined fracture grouting and GFRP soil nails ground improvement scheme is proposed by the authors as the alternative solution (Cheng *et al.* 2013), and the cost of the alternative scheme is critically reduced to 50% of the original scheme. In the past, fracture grouting was mainly adopted for compensation grouting, and the

Numerical Methods in Geotechnical Engineering

Abstract: Finite element method is the most popular and important numerical method in geotechnical engineering analysis and design. In this chapter, the basic formulation for finite element method will be introduced, followed by the various procedures in the actual implementation of the method which are illustrated by several sample Fortran programs. The style of Fortran programming, mesh generation, bandwidth/profile minimizer as well as the development of modern thick plate element are used for the illustration of the various techniques required in developing a finite element program.

Keyword: Fortran, Finite element, Isoparametric element, Programming style, Plate element, Shape functions.

2.1. INTRODUCTION TO PROGRAMMING

Many books on finite element method or numerical methods have provided sample programs for the readers to learn the actual implementation of the methods. Very good references are provided by Smith *et al.* (2014), Owen and Hinton (1980), Hinton and Owen (1977), Zienkiewicz *et al.* (2011). During learning the finite element method and the techniques in programming, the authors have learnt a lot from these references and many other technical papers. Many of these references on finite element method target at simple input data format and standard problems, but not much on other types of problems. Some open source programs or commercial programs are too fruitful with many libraries and functions which are not easy to modify or to learn. For example, the program Abaqus comes with series of program manuals which require a steep learning curve, and to develop a special add-in subroutine to Abaqus can be a tedious task. If a reader prefers to carry a new analysis and adopt his preferred pre-processor or post-processor, it is extremely time-consuming to spend the effort to learn some large scale commercial engineering programs. The present book actually targets at such a group of readers, which are most probably research students who need to develop computer codes for some new algorithms.

There are many numerical methods that are being used for various purposes, which include finite element method, boundary element method, distinct element

method, meshless method and others. Out of these methods, the boundary element method will not be discussed in this book, as there are many practical limitations to the boundary element method in general geotechnical engineering problems. Fortran language is adopted in this book, as many numerical programs are actually developed in Fortran because this language is targeted towards mathematical and matrix operations, and the relatively simple language format has avoided many bugs that may be found in C/C++. In views of that, the authors will only present the results in Fortran in this book, even though some engineering programs are developed in C by the authors.

2.1.1. Management of Input Data

As mentioned in chapter 1, most of the practical problems require the use of numerical methods and computer software for the analysis and design. Throughout the years, the authors have developed series of computer programs for teaching and research, and some of the experience and results will be shared in this book. To begin with, some skill commonly adopted by the authors are first discussed.

The authors have used Fortran 90/95 for the development of many structural and geotechnical programs. The authors have not adopted the more advanced Fortran 2003, 2008 or later Fortran versions, as Fortran 90/95 (Chapman, 2007) is adequate for most applications. There are some important utilities for which the authors adopt in various programs development, and some of these utilities will be discussed below. Throughout this book, the subroutines/programs are developed by mainly Lahey Fortran while GFortran and Absoft Fortran are also used for some cases, but the codes should also work under other Fortran compilers. The readers should be able to modify the codes to comply with different compilers, or to remove some old style programming formats. The authors assume the readers to have sufficient knowledge of basic Fortran language, and the works by Chapman (2007) and others can be referred if necessary.

Classically, Fortran, C other computer languages have only relatively simple methods in reading input files which are usually text file for ease of transportation between different systems. Even with the refined input standard in the latest Fortran or C language, the way a program reads the input file is still basically sequential, and a well-defined data structure is required. To allow for more flexible input approach, the authors have adopted the open source library FLIB with some modifications, and the revised version of FLIB can be obtained freely from the authors. FLIB adopts the operator overloading function available in Fortran 90/95, and only limited functions will be discussed in this section. With reference to the subroutine as shown below, the input module uses the functions

in FLIB through the modules STRPAK and FIOPAK, while the data required by the main program is communicated through module DATA.

SUBROUTINE INPUT

USE STRPAK ; USE FIOPAK ; USE DATA

Integer :: ndat, i, j, k, ndat, bore_n, bore_s, errar

Real :: Toler

Open (25, file='input.txt')

call vi_getvar(25,'1', 'TOLER', ndat, errvar)

if (ndat > 0) then ! toler is defined with a value in the input file

call vi_data(1, TOLER, errvar) ; CALL VI_ERASE

! read the value of toler as ndat > 0, else, do not read

endif

CALL VI_GETVAR(25,'1','bore_n', ndat,errar)

if (ndat > 0) then

CALL VI_DATA(1, bore_n, errvar) ; CALL VI_ERASE

endif

CALL VI_GETVAR(25,'1','bore_s',ndat,errar)

if (ndat > 0) then

CALL VI_DATA(1, bore_s, errvar) ; CALL VI_ERASE

endif

call vi_getvar(25,'1','bore_data',ndat,errvar)

if (ndat == bore_n*bore_s) then

allocate (bore(bore_n,bore_s))

k=0

CHAPTER 3**Plasticity, Limit Equilibrium and Limit Analysis Methods in Geotechnical Engineering**

Abstract: In this chapter, the ultimate limit state of a system is considered by means of limit equilibrium, plasticity slip line method, limit analysis and DLO methods. The basic plasticity formulation for the slip line method is given, which is applied to some classical geotechnical problems. Following this, the three major geotechnical problems are unified under the extremum principle by the plasticity formulation. There is also discussion on the basic formulation for the DLO method and the limitations of the method or the commercial program.

Keyword: Axi-symmetric, Bearing capacity, DLO, Extremum, Lateral earth pressure, Limit equilibrium, Limit analysis, Plasticity, Slip line, Slope stability.

3.1. INTRODUCTION TO ULTIMATE LIMIT STATE ANALYSIS

Due to the difficulties in defining the *in situ* stress, the complications in many constitutive models as well as the difficulty in determining the various parameters required for a constitutive model, many geotechnical analysis and design works are still based on the ultimate limit state consideration, despite many finite element programs with various capabilities developed over the years. This is not surprising for the engineers and researchers, particularly for the engineers. The authors are greatly interested in a case in Hong Kong, where a highly theoretical soil constitutive model was developed and calibrated for the various required parameters in laboratory. This model was used for the interpretation of a plate load test in Hong Kong, and the percentage error of the prediction was found to be around 400%. On the other hand, the model can predict extremely well for the laboratory test results. It is not surprising that many engineers have various hesitations on the use of many modern and sophisticated soil constitutive models. In Hong Kong and many other countries, these advanced constitutive models are not commonly used for practical purposes. Unless this critical limitation can be overcome, the very large gap between the theoretical development and the actual applications will remain there. Actually, Cheng has written a very complicated nonlinear large strain elasto-plastic geotechnical program with 13 constitutive mo-

dels, where different combinations of yield functions and plastic potential functions can be combined and used. So far, Cheng seldom adopts this program or other commercial programs using advanced constitutive model for real engineering design. The limited site investigations and laboratory/field test results for most projects cannot justify the choice of a highly sophisticated constitutive model, and many surprising results (usually local effects) can be obtained from every commercial program that the authors have tried when those advanced constitutive models are used. In fact, some engineers in Hong Kong will simply turn the complicated constitutive soil model to simple elastic/elasto-plastic model when numerical problems occur. Interestingly, many engineers find that a very famous geotechnical analysis program (the latest version already) can run into problems easily with the more advanced constitutive models for many excavation/lateral support problems. From the authors' view, a constitutive model which is not accurate enough for real problems or can run into numerical problems easily is not a good model, no matter how good is the theoretical background behind the model.

On the other hand, the use of the ultimate limit state for design is well-received and used by the engineers, with the application of a suitable factor of safety. With the experience accumulated over many years, engineers tend to rely more on the ultimate limit state analysis and design than the use of modern constitutive models for some types of problems. It is not surprising that some geotechnical designs are still based on the use of the limit state analysis up to the present. There many different references and research papers associated with this, and some additional references are given by Liu *et al.* (2019), Baars (2018), Nedderman (1992), Yu (2006), Davis and Selvadurai (2002), Rees (2006) as well as the classical works by Hill (1950), Sokolovskii (1965) and Chen (1975).

For stability analysis, there are various methods available to the engineers, and the choice of the method depends on the complexity of the geometry and the convenience in the solution. In this chapter, the slip line method, limit equilibrium method and limit analysis will be introduced for the lateral earth pressure, ultimate bearing capacity and slope stability problems. It is interesting to note that these three topics are usually considered separately in most of the books or research studies, and different methods of analyses have been proposed for individual problem even though they are governed by the same requirements for the ultimate conditions. Since the governing equations and boundary conditions for these problems are actually the same, Cheng and Li (2017) view that each problem can be viewed as the inverse of the other problems which will also be demonstrated in the present chapter. After the introduction of the three basic stability analysis methods, the unification of the three most important stability problems will be discussed.

The three stability methods together with the corresponding numerical solution techniques will be discussed with the use of different computer programs developed by the authors. The limit equilibrium methods as discussed in this chapter are available in the program SLOPE 2000 developed by Cheng, which can be obtained from the authors at natureymc@yahoo.com.hk. SLOPE2000 is also one of the analysis modules in the large scale geotechnical analysis and design package GEOCalc 1.0/2.0/4.0.

3.2. SLIP-LINE METHOD

At the ultimate condition, both equilibrium and yield conditions must be satisfied. Combining the Mohr-Coulomb yield criterion (which is generally adequate for soil) and the equilibrium equations, a set of hyperbolic partial differential equations of plastic equilibrium can be developed. In order to solve the governing partial differential equation, it is more convenient to transform the governing equations to curvilinear coordinates along the directions of the failure planes for mathematical convenience. Once the equations are solved, the failure modes with the corresponding systems of stresses will be automatically determined. The slip directions or slip lines constitute a network which is called slip-line field. The governing equations can be solved with adequate boundary conditions to investigate the stresses at the ultimate condition, and the solution of the problem is commonly taken as the rigorous solution, as the solutions are either similar to those from other methods or are better. Since the governing equations are written along the slip lines, the slip line fields corresponding to the solutions are commonly considered as the failure mechanism of the governing problem. For example, the bearing capacity of footing and the lateral earth pressure behind a retaining wall are commonly analyzed by the slip line analysis, but not for the slope stability problem.

Kötter (1903) was the first to derive the slip-line equations for two-dimensional ultimate problems, while Prandtl (1920) was the first to obtain an analytical solution for footing by assuming the weight of soil to be negligible. His results were then applied by Reissner (1924) and Novotortsev (1938) to different problems on the bearing capacity of footing on weightless soil. The inclusion of soil weight in the solution of the governing partial differential equation is analytically impossible, and Sokolovskii (1965) proposed a finite difference approximation of the slip-line equations for which the accuracy can be further improved by an iteration scheme (Cheng 2002, 2003), and such iteration to update the coordinates of the grid points on the slip line field has been demonstrated to be important for passive pressure evaluation. Sokolovskii (1965) solved many types of problems on the bearing capacity of footings, slopes as well as the lateral earth

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